

COMPITO DEL 18-12-09

Esercizio 1: Fermioni ideali in 2D

①

$$g(E) = \frac{2}{A} \sum_{\vec{p}} \delta(E - \frac{p^2}{2m})$$

$$= \frac{2}{A} \frac{A}{h^2} \int_0^\infty dp p \cdot 2\pi \delta(E - \frac{p^2}{2m})$$

$$= \frac{4\pi}{h^2} m \int_0^\infty d(\frac{p^2}{2m}) \delta(E - \frac{p^2}{2m})$$

$$\boxed{g(E) = \frac{4\pi m}{h^2} \vartheta(E) = \frac{m}{\pi h^2} \vartheta(E)}$$

$$\textcircled{2} \quad \frac{\langle N \rangle}{A} = \rho = \int_0^\infty dE \frac{g(E)}{e^{\beta(E-\mu)} + 1} = \frac{m k_B T}{\pi h^2} \int_0^\infty dt \frac{1}{\frac{e^t}{Z} + 1}$$

$$= \frac{2 \cdot 2\pi m k_B T}{h^2} \int_0^\infty dt z e^{-t} / (1 + z e^{-t})$$

$$= \frac{2}{\lambda^2} \int_0^Z d(z e^{-t}) / (1 + z e^{-t}) \quad y = z e^{-t}$$

$$\boxed{\rho = \frac{2}{\lambda^2} \ln(1+Z)} \quad \boxed{\beta \epsilon_F = \ln(1 + e^{\beta \mu})}$$

③

$$\beta P = \frac{\ln Z}{A} = \frac{2.57}{A\phi} \ln(1 + e^{-\beta(E_F - \mu)})$$

$$= \int_0^{\infty} dE g(E) \ln(1 + e^{-\beta(E - \mu)})$$

$$= \frac{4\pi m}{h^2} \int_0^{\infty} dE \ln(1 + e^{-\beta(E - \mu)})$$

$$= \frac{2}{\lambda^2} \int_0^{\infty} dt \ln(1 + ze^{-t}) = \frac{2}{\lambda^2} f_2(z)$$

$$\beta P = \frac{2}{\lambda^2} f_2(z); \quad f_2(z) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} z^n}{n^2}, \quad z < 1.$$

④ $\rho \lambda^2 \rightarrow 0 \quad \gamma = \lambda^2 \rho / 2$

$$Z = re^{\frac{\rho \lambda^2}{2}} - 1 \equiv e^{\gamma} - 1 = \gamma + \frac{\gamma^2}{2} + \frac{\gamma^3}{6} + \dots$$

$$\beta P = \frac{2}{\lambda^2} \left[Z - \frac{Z^2}{4} + \frac{Z^3}{9} + \dots \right]$$

$$= \frac{2}{\lambda^2} Z \left[1 - \frac{Z}{4} + \frac{Z^2}{9} + \dots \right]$$

$$= \frac{2}{\lambda^2} \gamma \left[1 + \frac{\gamma}{2} + \frac{\gamma^2}{6} + \dots \right] \left[1 - \frac{1}{4}(\gamma + \frac{\gamma^2}{2} + \dots) + \frac{\gamma^2}{9} + \dots \right]$$

$$= \rho \left[1 + \gamma \left(\frac{1}{2} - \frac{1}{4} \right) + \gamma^2 \left(-\frac{1}{8} + \frac{1}{6} - \frac{1}{9} + \frac{1}{9} \right) \right]$$

$$\frac{\beta P}{\rho} = 1 + \frac{\rho \lambda^2}{8} + \frac{1}{144} (\rho \lambda^2)^2 + \dots$$

Esercizio 2: Bosoni ideali in 4D

①

$$g(\epsilon) = \frac{1}{V} \sum_{\vec{p}} \delta(\epsilon - \frac{p^2}{2m})$$

$$= \frac{1}{V} \frac{V}{h^4} \frac{\pi^2}{2} \int_0^\infty dp p^3 \delta(\epsilon - \frac{p^2}{2m})$$

$$= \frac{\pi^2}{2h^4} \left[\frac{p^3}{p/m} \right]_{p=\sqrt{2m\epsilon}} = \frac{\pi^2 m}{2h^4} 2m\epsilon$$

$$= \frac{\pi^2 m^2}{h^4} \epsilon$$

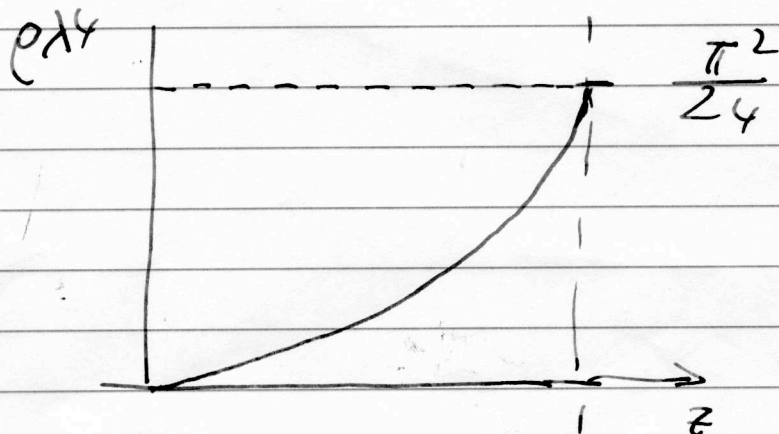
②

$$\frac{\langle N \rangle}{V} \equiv \rho_n = \int_0^\infty d\epsilon \frac{g(\epsilon)}{e^{\beta(\epsilon - \mu)} - 1} = \frac{\pi^2 m^2}{h^4} \int_0^\infty d\epsilon \frac{\epsilon}{\frac{e^{\beta\epsilon}}{z} - 1}$$

$$= \frac{\pi^2 m^2 (\frac{1}{\beta})^2}{h^4} \int_0^\infty dt \frac{t z e^{-t}}{1 - z e^{-t}} \quad \boxed{\rho = \rho_n + \rho_s}$$

$$\rho_n = \frac{1}{4\lambda^4} g_2(z) = \sum_{n=1}^\infty \frac{z^n}{n^2}; \quad \rho_s = \frac{1}{V} \frac{z}{1-z}$$

③



$$y_c = (\lambda^4 \rho)_c = \frac{\pi^2}{24}$$

$$\textcircled{4} \quad (\lambda^4 \rho)_c = \frac{g_2(1)}{4}$$

$$\rho_s = \rho - \rho_n = \rho - \frac{g_2(2)}{4\lambda^2}$$

$$\pi < \pi_c$$

$$\rho_s = \rho - \frac{g_2(1)}{4\lambda^2} = \rho - \frac{g_2(1)}{4\lambda^2 \rho} \rho$$

$$= \rho \left[1 - \frac{g_2(1)}{4\lambda^2 \rho} \right] = \rho \left[1 - \frac{\lambda^4 \rho_k}{\lambda^4 \rho} \right]$$

$$= \rho \left[1 - \left(\frac{\pi}{\pi_c} \right)^2 \right]$$

$$\boxed{\frac{\rho_s}{\rho} = 1 - \left(\frac{\pi}{\pi_c} \right)^2}$$