Smoke rings in atmospheric science

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(Dated: November 14, 2025)

Extract from the course 302 “Atmospherical Science” by Walter Robinson held at Urbana/Champaign during Fall 1992 [1].

In 2007 the Nobel Peace Prize was awarded to the Intergovernmental Panel on Climate Change (IPCC) and former US Vice President Al Gore Jr., the Norwegian Nobel Committee called special attention to their efforts to obtain and disseminate greater knowledge concerning man-made climate changes and the steps that need to be taken to counteract those changes. According to the IPCC, there is a real danger that the climate changes may also increase the danger of war and conflict, because they will place already scarce natural resources, not least drinking water, under greater pressure and put large population groups to flight from drought, flooding, and other extreme weather conditions [2].

Keywords: Atmospheric science, circulation, smoke ring

I. INTRODUCTION

In *dynamic meteorology* one applies the fundamental principles of physics to understand and predict the *motion* of the atmosphere. Focusing on the motions of the atmosphere, this subject does *not* address the interactions of electromagnetic radiation or the chemistry with the atmosphere or the microscopic processes involved in the formation of precipitation [3].

Consider a velocity profile $\boldsymbol{u}$ of the atmosphere. Its *circulation* is defined as [4]

$$C = \oint \boldsymbol{u} \cdot d\boldsymbol{\ell}, \tag{1.1}$$

and the *vorticity* is the circulation for unit area.

The time derivative of the circulation

$$\frac{dC}{dt} = \frac{d}{dt}\oint \boldsymbol{u} \cdot d\boldsymbol{\ell} = \oint \left[\frac{d\boldsymbol{u}}{dt} \cdot d\boldsymbol{\ell} + \boldsymbol{u} \cdot d\boldsymbol{u}\right] = \oint \frac{d\boldsymbol{u}}{dt} \cdot d\boldsymbol{\ell}. \tag{1.2}$$

To study the dynamics of *smoke rings* we may use the momentum equation in an inertial frame and neglect friction (assuming a negligible viscosity)

$$\frac{d\boldsymbol{u}}{dt} = -g\hat{\boldsymbol{z}} - \frac{\boldsymbol{\nabla} P}{\rho}, \tag{1.3}$$

where g is the gravity constant, ρ is the density of the air, and P is the atmospheric pressure. We then find

$$\frac{dC}{dt} = -\oint \frac{\boldsymbol{\nabla} P}{\rho} \cdot d\boldsymbol{\ell} = -\oint \frac{dP}{\rho}. \tag{1.4}$$

In an incompressible fluid ρ is constant and we find that $dC/dt = 0$, i.e. C is a constant. This is also known as Kelvin circulation theorem that holds more generally for a fluid with $P = P(\rho)$ or $\rho = \rho(P)$.

In Fig. 1 we show a vertical section cutting a smoke ring along its diameter and the velocity profile of the air. From the close up of Fig. 2 we can say that $C = 2\pi r\bar{u}$ with r the internal radius of the smoke torus and $\bar{u}$ the air velocity at the torus surface. Since C is constant as the smoke ring evolves in time its final fate is to disappear with $R \to \infty$, the ring radius, and $r \to 0$, the torus internal radius. So that we conclude that the speed of the air at the torus surface must

$$\bar{u} \longrightarrow \infty. \tag{1.5}$$

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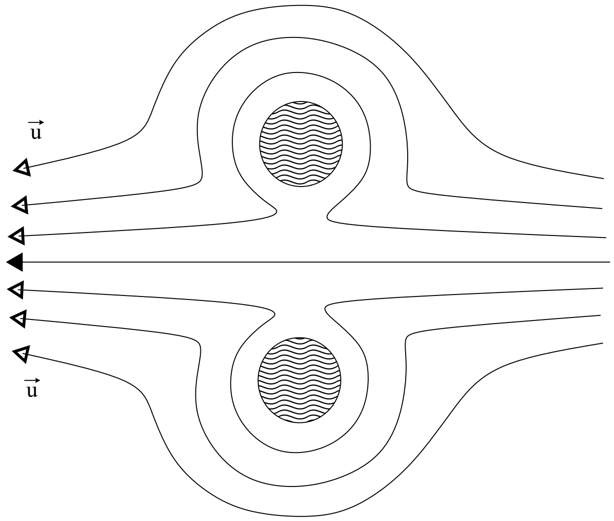

FIG. 1. Vertical section cutting a smoke ring along its diameter and the velocity profile of the air. The smoke is in the wavy regions.

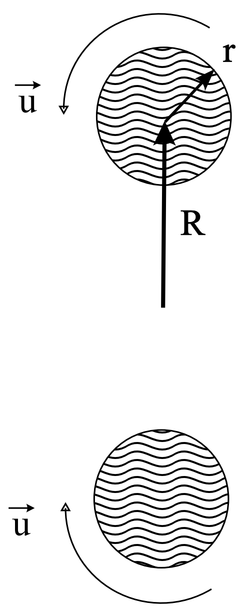

FIG. 2. Close up of the vertical section cutting a smoke ring along its diameter. R is the outer radius of the smoke ring torus and r its inner radius. The velocity $\boldsymbol{u}$ at the torus surface has magnitude $\bar{u}$.

AUTHOR DECLARATIONS

Conflicts of interest

None declared.

Data availability

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Funding

None declared.

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[3] R. Fantoni, Two-phase coexistence for hydrogen-helium mixtures, Phys. Rev. E **92**, 012133 (2015).
[4] L. D. Landau and E. M. Lifshitz, *Fluid mechanics*, Course of Theoretical Physics, Vol. 6 (Butterworth Heinemann, 1959) translated from the Russian by J. B. Sykes and M. J. Kearsley, edited by E. M. Lifshitz and L. P. Pitaevskii.