Temperature of the Vacuum

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In a recent trilogy we proposed a Statistical Theory of General Relativity spacetime. Here we apply our new theory to determine the (energy) "density" and (virial) "temperature" dependence of the structure of the spacetime quantum vacuum working on the simple case of a real massless scalar field in a local Lorentz frame.

Keywords: Quantum Vacuum; General Relativity; Temperature; Pair Correlation Function; Structure

I. INTRODUCTION

In a recent trilogy [1–3] we proposed a Statistical Theory of Gravity. This allowed us to determine a "virial temperature" of the spacetime metric tensor field. Albeit still under refinement, the theory is already able to offer some measurable predictions. In fact, as we will see in this work, it influences the energy density *structure* of the spacetime vacuum. Unfortunately, with the current equipment we are unable to directly measure these structure variations due to the temperature. But we can hope in some indirect observations of the consequences on the Hubble rate of expansion of the Universe, the parameter $H = \dot{a}/a$, where $a > 0$ is the scale factor which enters the spatial components of the cosmological metric tensor field and is proportional to the average separation between objects, such as galaxies, and the dot denotes a derivative with respect to the cosmological time. As Edwin Hubble discovered in 1929 the parameter H is a measurable quantity. For example the current Hubble parameter, the *Hubble constant*, is estimated to be $H_0 \approx 7\%/\mathrm{Gyr}$. Hubble constant is made of two contributions: a gravitational one and one due to the cosmological constant. Wheeler's spacetime foam [4, 5] suggests that a foamy structure leads to the cosmological constant we see today. Cosmological models for the metric tensor field began with the one of Friedmann-Lemaître-Robertson-Walker and were refined in various ways [6] in order to take care of the inhomogeneity and anisotropy of spacetime predicted by a quantum vacuum. These calculations have macroscopic consequences at the level of the description of the Universe evolution. In the sense that $a(t)$ can have various different functional forms: respect to the current situation it can remain constant, it can grow exponentially or with other laws, or it can even bend downwards with some law. Through the Universe exploration we can hope to be able to at least have some indirect insight on the effect of temperature on the spacetime vacuum. One crucial step in our formulation is assuming that the vacuum energy density and its temperature are constant in cosmological time and uniform in cosmological space. In this respect the energy density of the spacetime vacuum ρ_{vacuum} is conceptually different from the energy density of matter ρ_{matter} in the Universe: while the matter mass has to be considered a constant during the Universe evolution so that $\rho_{\mathrm{matter}} \propto a(t)^{-3}$ the vacuum is created or destroyed during the Universe expansion or contraction with ρ_{vacuum} kept constant. In other words the spacetime vacuum behaves like a fluid occupying a larger or smaller volume but keeping its density constant. So it behaves like a cosmological constant in the Universe evolution. We may consider this fluid as the source of dark energy.

Our virial temperature is conceptually different from the Davies-Unruh [7, 8] local temperature. The latter is in fact defined as $\mathscr{T}_{DU} = \hbar g/2\pi c k_B \approx 4.06 \times 10^{-21}\ \mathrm{Ks^2m^{-1}} \times\ g$ where g is the proper uniform acceleration of a detector in vacuum. Therefore while our virial temperature is a gravitational one ascribed to the spacetime by the stress-energy tensor, the one of Davies-Unruh is not, it cannot be derived from the Einstein field equations since the detector is not following a geodetic of the spacetime.

The fluids in nature (photon liquid, electron liquid, neutron liquid, ...) carry a temperature which through the stress-energy tensor determines the "virial temperature" of spacetime [1] which in turn excites the pure state of the quantum vacuum stimulating particle-antiparticle production and recombination. We can then talk of the temperature of the quantum vacuum of spacetime.

We will now first discuss about the structural properties of the quantum vacuum for a real massless scalar field permeating the spacetime of a Local Lorentz Frame (LLF) and later extend our discussion to the case of General Relativity (GR). From our recent work [1] follows that $\langle R\rangle_\beta \approx 16\pi G\mathscr{T}/c^4\bar{v}$ where the term on the left hand side is the thermal average of R, the Ricci curvature scalar, and, on the right hand side, G is Newton universal gravitation constant, c is the speed of light, $\mathscr{T}$ is our "virial temperature" of spacetime and $\bar{v}$ is a constant carrying the dimensions

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of length squared divided by energy. So in flat space $R=0$, $\mathscr{T}=0$ and the Minkowsky spacetime quantum vacuum structure will not depend on temperature. On the other hand in GR we will be able to determine an approximation that takes care of the dependence of the vacuum spacetime structure from our temperature $\mathscr{T}$.

II. DISCUSSION

In Ref. [6] the simple case of a real massless scalar field in flat Minkowski spacetime is considered [1]

$$\phi(t,\boldsymbol{r})=\int\frac{d\boldsymbol{k}}{(2\pi)^{3/2}\sqrt{2\omega}}\left[a_{\boldsymbol{k}}e^{-i(\omega t-\boldsymbol{k}\cdot\boldsymbol{r})}+a_{\boldsymbol{k}}^\dagger e^{i(\omega t-\boldsymbol{k}\cdot\boldsymbol{r})}\right], \tag{2.1}$$

where the temporal frequency ω and the spatial frequency $\boldsymbol{k}$ are related by $\omega=|\boldsymbol{k}|$ in natural units $\hbar=c=1$.

The vacuum pure state $|0\rangle$ is defined by

$$a_{\boldsymbol{k}}|0\rangle=0\quad\forall\boldsymbol{k} \tag{2.2}$$

and $a_{\boldsymbol{k}}^\dagger|0\rangle=|\omega,\boldsymbol{k}\rangle$ with $\langle\omega,\boldsymbol{k}'|\omega,\boldsymbol{k}\rangle=\delta(\boldsymbol{k}-\boldsymbol{k}')$ so that the vacuum expectation value of the square modulus of the field, $\langle\phi^2(t,\boldsymbol{r})\rangle_0=\langle 0|\phi^2(t,\boldsymbol{r})|0\rangle=\Lambda^2/8\pi^2$ with $|\boldsymbol{k}|=\Lambda$ a high-energy (ultraviolet) cutoff. In Eq. (2.1) the first term creates an antiparticle in the sense of Dirac and the second a particle.

The vacuum state is an eigenstate of the Hamiltonian $\mathscr{H}=\int d\boldsymbol{r}\,T_{00}=\frac{1}{2}\int d\boldsymbol{k}\omega\,(a_{\boldsymbol{k}}a_{\boldsymbol{k}}^\dagger+a_{\boldsymbol{k}}^\dagger a_{\boldsymbol{k}})$, where $T_{00}(t,\boldsymbol{r})=\frac{1}{2}[\dot{\phi}^2+(\boldsymbol{\nabla}\phi)^2]$. But it is *not* an eigenstate of the energy density T_{00}. This fact gives rise to a non trivial vacuum structure. Direct calculation (See appendix A of Ref. [6]) shows that the pair correlation function

$$g(x,x')=1-\frac{\rho^{(2)}(x,x')}{\frac{2}{3}\rho^2}, \tag{2.3}$$

where $x=(x^0,x^1,x^2,x^3)=(t,\boldsymbol{r})$, $x'=(t',\boldsymbol{r}')$ are two spacetime events and

$$\rho=\rho^{(1)}(x)=\langle T_{00}(x)\rangle_0, \tag{2.4a}$$

$$\text{covariance}(\rho)=\rho^{(2)}(x,x')=\langle\{[T_{00}(x)-\rho^{(1)}(x)][T_{00}(x')-\rho^{(1)}(x)]\}\rangle_0. \tag{2.4b}$$

where $\{AB\}=\frac{1}{2}(AB+BA)$ for any two operators A and B, and a simple calculation shows that $\rho=\Lambda^4/16\pi^2$ is a constant over spacetime and can be considered as the energy "density" of spacetime vacuum. After a lenghty calculation we [2] find the following result

$$\rho^{(2)}(x,x')=\frac{1}{2}\int\frac{d\boldsymbol{k}\,d\boldsymbol{k}'}{(2\pi)^6}\frac{(\omega\omega'-\boldsymbol{k}\cdot\boldsymbol{k}')^2}{2\omega 2\omega'}\cos[(\omega-\omega')\Delta t-(\boldsymbol{k}+\boldsymbol{k}')\cdot\Delta\boldsymbol{r}], \tag{2.5}$$

where $\Delta t=t-t'$ and $\Delta\boldsymbol{r}=\boldsymbol{r}-\boldsymbol{r}'$.

As can be seen from Fig. 1 the pair correlation function of (2.3) reveals an inhomogeneous and anisotropic spacetime vacuum. It can also be easily shown that $\rho^{(2)}(x,x)=\frac{2}{3}\rho^2$ so that $g(0)=0$ which can be pictured as a spacetime vacuum *hole* at events contact and on the other hand $g\to 1$ at large events separation which can be interpreted as a decorrelation among spacetime events of the vacuum which becomes *uniform* and *isotropic* on a large spacetime scale. From the figure we see how both the time like pair correlation function at $|\boldsymbol{r}-\boldsymbol{r}'|=0$ and the space like one for $t-t'=0$ grow monotonously towards the uniform and isotropic spacetime at large events separation.

In this picture there is no space left for a "temperature" of the vacuum. Instead we expect the structure of the spacetime vacuum to feel and depend on temperature too.

In a recent work [1] we introduced and defined a "virial temperature" of General Relativity (GR) spacetime. Aim of the present work is to determine how that temperature can affect the structure of the quantum vacuum of spacetime.

Note that the result of Eq. (2.3) and Fig. 1 looses any value in GR. In fact the covariance of Eq. (2.4b) is inherently non local and its calculation in a LLF will not be useful in GR.

Now in GR the stress-energy tensor for the massless scalar field ϕ becomes, for a generic spacetime metric $g_{\mu\nu}$,

$$T_{\mu\nu}=\phi_{,\mu}\phi_{,\nu}^\dagger-\frac{1}{2}g_{\mu\nu}\phi^{,\alpha}\phi_{,\alpha}^\dagger \tag{2.6}$$

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[1] Note that the integral measure here is chosen to be not Lorentz invariant in order to simplify the later structure calculation of the vacuum.

[2] We found a sign error in their Eq. (A3).

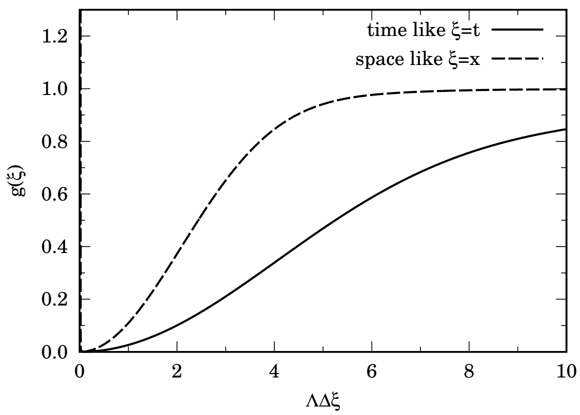

FIG. 1. The LLF pair correlation function $g(\xi)$ of Eq. (2.3): when the separation of the events x and x' is time-like for $\boldsymbol{r} = \boldsymbol{r}'$, $\Delta\xi = |t - t'|$ and when it is space-like for $t = t'$, $\Delta\xi = |\boldsymbol{r} - \boldsymbol{r}'|$.

where a comma stands for a partial derivative and we allow the field to be complex for the sake of more generality.

According to Einstein field equations the stress-energy tensor of the scalar field will induce a curvature of the spacetime

$$\langle G_{\mu\nu}\rangle_0 = 8\pi\langle T_{\mu\nu}\rangle_0, \tag{2.7}$$

where $G_{\mu\nu}$ is Einstein tensor and we are using Planck natural units $\hbar = c = G = k_B = 1$. Once again $\langle\ldots\rangle_0$ stands for a quantum vacuum expectation value $\langle 0|\ldots|0\rangle$. In our previous work [1] we defined the most natural thermal average for spacetime that we will here indicate with the notation $\langle\ldots\rangle_\beta$ where $\beta = 1/\mathscr{T}$ is the inverse *temperature* [3]. We will then more correctly need to average Eq. (2.7) like so

$$\langle\langle G_{\mu\nu}\rangle_0\rangle_\beta = 8\pi\langle\langle T_{\mu\nu}\rangle_0\rangle_\beta. \tag{2.8}$$

Note that while the thermal average $\langle\ldots\rangle_\beta$ acts only on the spacetime metric $g_{\mu\nu}$ the vacuum expectation value $\langle\ldots\rangle_0$ acts only on the field on the right hand side of Eq. (2.7). On the left it has no effect and we can then rewrite

$$\langle G_{\mu\nu}\rangle_\beta = 8\pi\langle\langle T_{\mu\nu}\rangle_0\rangle_\beta. \tag{2.9}$$

For example for the energy density we will find

$$\langle\langle T_{00}\rangle_0\rangle_\beta = \langle|\dot{\phi}|^2\rangle_0 - \frac{1}{2}\langle g_{00}\, g_{\mu\nu}\rangle_\beta\langle\phi^\mu_{,}\phi^{\dagger\nu}_{,}\rangle_0, \tag{2.10}$$

where as usual there is a hidden summation over repeated lower and upper indexes.

In Ref. [1] we were also able to render explicit the temperature. One simply has to trace out the stress-energy tensor like so

$$\mathscr{T} = -\frac{\bar{\upsilon}}{4}\langle T^\mu_\mu\rangle_\beta, \tag{2.11}$$

where $T^\mu_\mu = -g_{\mu\nu}\phi^\mu_{,}\phi^{\dagger\nu}_{,}$ is the stress-energy tensor trace. Taking a vacuum expectation value of this expression we find

$$\mathscr{T} = \frac{\bar{\upsilon}}{4}\langle g_{\mu\nu}\rangle_\beta\langle\phi^\mu_{,}\phi^{\dagger\nu}_{,}\rangle_0. \tag{2.12}$$

We will then redefine the first two $n-$points energy density correlation functions, now in GR

$$\rho = \rho^{(1)}(x) = \langle\langle T_{00}(x)\rangle_0\rangle_\beta, \tag{2.13a}$$

$$\text{covariance}(\rho) = \rho^{(2)}(x,x') = \langle\langle\{[T_{00}(x) - \rho^{(1)}(x)][T_{00}(x') - \rho^{(1)}(x)]\}\rangle_0\rangle_\beta, \tag{2.13b}$$

[3] In Planck units the spacetime temperature $\mathscr{T}$ varies on Planck energy scale $\sqrt{\hbar c^5/G} = 1.9561\times 10^9$ J.

which extend Eqs. (2.4a)-(2.4b) to full GR. We will assume that the field can still be written as in Eq. (2.1). We can then again easily calculate the two vacuum expectation values in Eq. (2.10)

$$\langle|\dot{\phi}|^2\rangle_0 = \frac{\Lambda^4}{16\pi^2}, \tag{2.14}$$

$$\langle\phi^\mu_{,}\phi^{\dagger\nu}_{,}\rangle_0 = \int \frac{d\boldsymbol{k}}{(2\pi)^3}\frac{p^\mu p^\nu}{2p^0}, \tag{2.15}$$

where $p = (p^0, p^1, p^2, p^3) = (\omega, \boldsymbol{k})$ is the four momentum. But we will now follow a different route. We will make the following approximation in Eq. (2.10)

$$\langle g_{00}\, g_{\mu\nu}\rangle_\beta \approx \langle g_{00}\rangle_\beta\langle g_{\mu\nu}\rangle_\beta \tag{2.16}$$

which allows to use the result of Eq. (2.12) to find

$$\rho = \langle|\dot{\phi}|^2\rangle_0 - \frac{2\mathscr{T}}{\bar{\upsilon}}\langle g_{00}\rangle_\beta. \tag{2.17}$$

Assuming furthermore that

$$\frac{\langle g_{00}\rangle_\beta}{\bar{\upsilon}} \approx \kappa, \tag{2.18}$$

a constant independent from temperature, we finally reach the following result [4]

$$\rho = \frac{\Lambda^4}{16\pi^2} - 2\kappa\mathscr{T}, \tag{2.19}$$

where the two approximations (2.16) and (2.18) follow a mean tensor field spirit.

Repeating now the calculation carried on for the LLF we now find from Eq. (2.13b)

$$\rho^{(2)}(x, x') = -\int \frac{d\boldsymbol{k}\,d\boldsymbol{k}'}{(2\pi)^6}\frac{\left(\omega\omega' - \frac{1}{2}\langle g_{00}\rangle_\beta\langle g_{\mu\nu}\rangle_\beta p^\mu p'^\nu\right)^2}{2\omega 2\omega'}\cos[(\omega-\omega')\Delta t - (\boldsymbol{k}-\boldsymbol{k}')\cdot\Delta\boldsymbol{r}], \tag{2.20}$$

which correctly reduces to (2.5) when $g_{\mu\nu} = \eta_{\mu\nu} = \mathrm{diag}\{-1, 1, 1, 1\}$.

We can further think about a third approximation in order to make some progress towards an insight on the pair correlation function of the quantum vacuum of full GR spacetime. a first guess could be for example the following

$$\rho^{(2)}(x, x') \approx \int \frac{d\boldsymbol{k}\,d\boldsymbol{k}'}{(2\pi)^6}\frac{\left[\omega\omega' - 2\bar{\kappa}\mathscr{T}\boldsymbol{k}\cdot\boldsymbol{k}'/(kk')\right]^2}{2\omega 2\omega'}\cos[(\omega-\omega')\Delta t - (\boldsymbol{k}+\boldsymbol{k}')\cdot\Delta\boldsymbol{r}], \tag{2.21}$$

where $\bar{\kappa}$ is a constant of dimension of energy. The pair correlation function,

$$g(x, x') = 1 - \frac{\rho^{(2)}(x, x')}{\rho^{(2)}(x, x)}, \tag{2.22}$$

$$\rho^{(2)}(x, x) \approx \left(\frac{\Lambda^4}{16\pi^2}\right)^2 + \frac{\Lambda^4(\bar{\kappa}\mathscr{T})^2}{48\pi^4}. \tag{2.23}$$

is shown in Fig. 2. From the figure we see how at low temperature the temporal structure of the spacetime quantum vacuum starts oscillating around the uniform and isotropic large separation limit. On the other hand the spatial structure remains monotonic. We then see how GR allows for a "density" and "temperature" dependence of the spacetime quantum vacuum. Notwithstanding the three approximations made and the assumption on the functional form of the scalar field our final result could be of some value as an application of our Statistical Theory of Gravity [1–3].

III. CONCLUSIONS

In this brief paper we showed how the unification of the Theory of General Relativity and of Statistical Physics allows to treat the quantum vacuum as a spacetime "fluid" with a structure which depends on the (energy) density and on our [1–3] (virial) temperature.

This "fluid" could offer a clue to the search for the *dark energy* which we today think is the missing ingredient necessary to explain experimental observations of our cosmos.

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[4] Note that our virial temperature is a local quantity which can only [1] depend on space, so $\mathscr{T} = \mathscr{T}(\boldsymbol{r})$ in the most general case. In a cosmological model we will assume it to be uniform throughout the whole Universe.

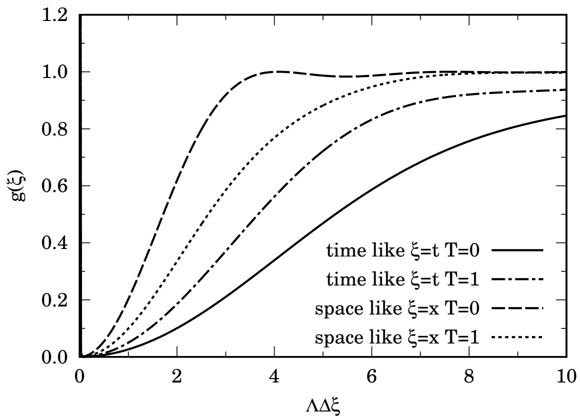

FIG. 2. The GR pair correlation function $g(\xi)$ of Eq. (2.22) at two temperatures $\mathscr{T} = 0, 1$: when the separation of the events x and x' is time-like for $\boldsymbol{r} = \boldsymbol{r}'$, $\Delta\xi = |t - t'|$ and when it is space-like for $t = t'$, $\Delta\xi = |\boldsymbol{r} - \boldsymbol{r}'|$.

AUTHOR DECLARATIONS

Conflicts of interest

None declared.

Data availability

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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