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Statistical Gravity Through Affine Quantization

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Abstract: I propose a possible way to introduce the effect of temperature (defined through the virial theorem) into Einstein's theory of general relativity. This requires the computation of a path integral on a ten-dimensional *flat* space in a four-dimensional spacetime *lattice*. Standard path integral Monte Carlo methods can be used to compute this.

Keywords: general relativity; Einstein–Hilbert action; statistical physics; path integral; Monte Carlo; virial theorem

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1. Introduction

A still unsolved problem in physics is the formulation of a well -defined theory unifying gravity and quantum mechanics. In this paper, we propose a possible way to estimate thermal statistical effects on the fabric of Einstein's spacetime. Once an action for the Einstein field equations of general relativity is found, we can use it to construct a path integral formulation for statistical gravity which may able to describe quantum effects at low temperatures. Recent progress on successful affine quantization [1] of euclidean relativistic scalar field theories may be important in making this path integral mathematically well-defined and numerically accessible, for example, through the usual path integral Monte Carlo method [2].

Some recent developments on gravity-quantum physics(statistical physics) interplay include the following:

On 15 August 2016 Jeff Steinhauer, an experimental physicist at the Technion-Israel Institute of Technology in Haifa, created an artificial black hole that seemed to emit 'Hawking radiation' on its own, from quantum fluctuations that emerge from its experimental set-up [3]. Using a Bose–Einstein condensate (BEC) of **rubidium cold atoms,** he created an event horizon by accelerating the atoms until some were travelling at more than 1 mm/s—a supersonic speed for the condensate. On one side of their acoustical event horizon, where the atoms move at supersonic speeds, phonons became trapped. Furthermore, when Steinhauer took pictures of the BEC, he found correlations between the densities of atoms that were at an equal distance from the event horizon but on opposite sides. This demonstrates that pairs of phonons were entangled – a sign that they originated spontaneously from the same quantum fluctuation and that the BEC was producing Hawking radiation.

Although quantum objects, like **the proton**, are not conventionally considered within a unified physics framework that includes gravity, it is significant to note that it has recently been discovered that these particles are more dense than massively compact objects like neutron stars. Scientists at the Jefferson Lab have measured pressures of 100 billion quintillion pascals at the proton's core—about ten times greater than the pressure inside neutron stars. These are the same pressure gradients computed by Haramein et al. [4], from first-principle considerations alone, on quantum vacuum fluctuations and gravitational effects at the proton scale, suggesting spacetime curvature alone is the source of the mass and force. The measurements by the Jefferson Lab team were achieved through innovative experiments that used pairs of photons to simulate gravitational interactions, allowing researchers to map the internal forces and pressures of the proton for the first time. The team found two distinct regions of specific pressures, like two phases within the proton, which were nearly exactly described in the study "The Origin of Mass and the Nature of Gravity" [4] by Haramein and the research team at the International Space Federation. This is a perfect example of how empirical research can reveal when a theory is on the right track, like when the Muonic measurements of the proton radius confirmed Haramein's prediction of a smaller proton radius than what was expected by the Standard Model, and now, experimental data is revealing that, indeed, the mass-energy and pressures within the proton are sufficient and that spacetime curvature must be a major consideration in the mass and binding forces that are observed for nucleons.

In 1974, Stephen Hawking predicted that quantum effects in the proximity of a black hole lead to the emission of particles and black hole evaporation. At the very heart of this process lies a logarithmic phase singularity which leads to the Bose–Einstein statistics of Hawking radiation. An identical singularity appears in the elementary quantum system of the inverted harmonic oscillator. In Ref. [5], the authors report the observation of the onset of this logarithmic phase singularity emerging at a horizon in phase space and giving rise to a Fermi–Dirac distribution. For this purpose, they utilize **solitonic surface gravity water waves** and freely propagated an appropriately tailored energy wave function of the inverted harmonic oscillator to reveal the phase space horizon and the intrinsic singularities. Due to the presence of an amplitude singularity in this system, the analogous quantities display a Fermi–Dirac rather than a Bose–Einstein distribution.

In this work, we explore the extreme consequences of the belief that the *Wick rotation*, which allows us to go back and forth between quantum physics and statistical physics in a non-general relativity framework, will continue to do so in a (quantum) general relativity framework as well. This belief is based only on the grounds of a mathematical extension, but it allows us to reach the amusing physical interpretation of a fabric of space–time (the main actor of the new theory) which has itself thermal fluctuations. We will in fact introduce an effective temperature (field) and we will show its relationship with the absolute temperature of thermodynamics in the non-quantum non-relativistic limit.

2. Einstein's Field Equations from a Variational Principle

Sempre caro mi fu quest'ermo colle,
e questa siepe, che da tanta parte
dell'ultimo orizzonte il guardo esclude.
Giacomo Leopardi "L' Infinito"

The Einstein–Hilbert action in general relativity is the action that yields the Einstein field equations through the stationary-action principle. With the $(- + + +)$ metric signature, the gravitational part of the action is given as follows [6,7]:

$$S = \frac{1}{2\kappa} \int R\sqrt{-g}\, \mathrm{d}^4x, \tag{1}$$

where $g = \det(g_{\mu\nu})$ is the determinant of the metric tensor matrix, $\sqrt{-g}$ is the scalar density, $\sqrt{-g}\, \mathrm{d}^4x$ is the invariant volume element, R is the Ricci scalar, and $\kappa = 8\pi G c^{-4}$ is the Einstein gravitational constant (G is the gravitational constant and c is the speed of light in vacuum). If it converges, the integral is taken over the whole spacetime. If it does not converge, S is no longer well-defined, but a modified definition where one integrates over arbitrarily large, relatively compact domains, still yields the Einstein equation as the Euler–Lagrange equation of the Einstein–Hilbert action. The action was proposed [6] by David Hilbert in 1915 as part of their application of the variational principle to a combination of gravity and electromagnetism.

The stationary-action principle then tells us that to recover a physical law, we must demand that the variation of this action with respect to the inverse metric be zero, yielding the following:

$$\begin{aligned} 0 = \delta S &= \int \left[\frac{1}{2\kappa} \frac{\delta(\sqrt{-g}R)}{\delta g^{\mu\nu}} \right] \delta g^{\mu\nu} \, \mathrm{d}^4 x && (2) \\ &= \int \left[\frac{1}{2\kappa} \left(\frac{\delta R}{\delta g^{\mu\nu}} + \frac{R}{\sqrt{-g}} \frac{\delta \sqrt{-g}}{\delta g^{\mu\nu}} \right) \right] \delta g^{\mu\nu} \sqrt{-g} \, \mathrm{d}^4 x. && (3) \end{aligned}$$

Since this equation should hold for any variation of $\delta g^{\mu\nu}$, it implies that

$$\frac{\delta R}{\delta g^{\mu\nu}} + \frac{R}{\sqrt{-g}} \frac{\delta \sqrt{-g}}{\delta g^{\mu\nu}} = 0 \tag{4}$$

is the equation of motion for the metric field.

The variation of the Ricci scalar in Equation (4) follows from varying the Riemann curvature tensor, and then the Ricci curvature tensor. The first step is captured by the Palatini identity as follows:

$$\delta R_{\sigma\nu} \equiv \delta R^{\rho}{}_{\sigma\rho\nu} = \left(\delta \Gamma^{\rho}_{\nu\sigma} \right)_{;\rho} - \left(\delta \Gamma^{\rho}_{\rho\sigma} \right)_{;\nu}. \tag{5}$$

Using the product rule, the variation of the Ricci scalar $R = g^{\sigma\nu} R_{\sigma\nu}$ then becomes,

$$\begin{aligned} \delta R &= R_{\sigma\nu} \delta g^{\sigma\nu} + g^{\sigma\nu} \delta R_{\sigma\nu} \\ &= R_{\sigma\nu} \delta g^{\sigma\nu} + \left(g^{\sigma\nu} \delta \Gamma^{\rho}_{\nu\sigma} - g^{\sigma\rho} \delta \Gamma^{\mu}_{\mu\sigma} \right)_{;\rho}, \end{aligned} \tag{6}$$

where we also used the metric compatibility $g^{\mu\nu}_{;\sigma} = 0$, and renamed the summation indices $(\rho, \nu) \to (\mu, \rho)$ in the last term. When multiplied by $\sqrt{-g}$, the term $\left(g^{\sigma\nu} \delta \Gamma^{\rho}_{\nu\sigma} - g^{\sigma\rho} \delta \Gamma^{\mu}_{\mu\sigma} \right)_{;\rho}$ becomes a total derivative, since for any vector A^{λ} and any tensor density $\sqrt{-g}\, A^{\lambda}$, we have

$$\sqrt{-g}\, A^{\lambda}_{;\lambda} = \left(\sqrt{-g}\, A^{\lambda} \right)_{;\lambda} = \left(\sqrt{-g}\, A^{\lambda} \right)_{,\lambda}. \tag{7}$$

By Stokes' theorem, this only yields a boundary term when integrated. The boundary term is in general non-zero, because the integrand depends not only on $\delta g^{\mu\nu}$ but also on its partial derivatives $\partial_{\lambda}\, \delta g^{\mu\nu} \equiv \delta\, \partial_{\lambda} g^{\mu\nu}$. However, when the variation of the metric $\delta g^{\mu\nu}$ vanishes in a neighbourhood of the boundary, or when there is no boundary, this term does not contribute to the variation of the action. Thus, we can forget about this term and simply obtain

$$\frac{\delta R}{\delta g^{\mu\nu}} = R_{\mu\nu}. \tag{8}$$

at events not in the closure of the boundary.

The variation of the determinant in Equation (4) requires Jacobi's formula, and the rule for differentiating a determinant is as follows:

$$\delta g = g g^{\mu\nu} \delta g_{\mu\nu}. \tag{9}$$

Using this we get

$$\delta \sqrt{-g} = -\frac{1}{2\sqrt{-g}} \delta g = \frac{1}{2} \sqrt{-g} \left(g^{\mu\nu} \delta g_{\mu\nu} \right) = -\frac{1}{2} \sqrt{-g} \left(g_{\mu\nu} \delta g^{\mu\nu} \right) \tag{10}$$

In the last equation we used the fact that from the symmetry of the metric tensor and $g_{\mu\nu}g^{\nu\mu} = \delta^{\mu}_{\mu} = 4$ follows

$$g_{\mu\nu}\delta g^{\mu\nu} = -g^{\mu\nu}\delta g_{\mu\nu} \tag{11}$$

Thus, we conclude that

$$\frac{1}{\sqrt{-g}}\frac{\delta\sqrt{-g}}{\delta g^{\mu\nu}} = -\frac{1}{2}g_{\mu\nu}. \tag{12}$$

Now that we have all the necessary variations at our disposal, we can insert Equations (12) and (8) into the equation of motion (4) for the metric field to obtain

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 0, \tag{13}$$

which is the Einstein field equation in vacuum.

Moreover, since Einstein's tensor $G_{\mu\nu}$ appears from a variational principle as follows:

$$\frac{\delta S}{\delta g^{\mu\nu}} = \frac{1}{2\kappa}\sqrt{-g}\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right) = \frac{1}{2\kappa}G_{\mu\nu}, \tag{14}$$

its covariant divergence is necessarily zero [7].

Matter or electromagnetic fields will produce a curvature of spacetime. In order to take this into account, it is necessary to add a term $\mathcal{L}_F$ as follows:

$$S = \int\left(\frac{1}{2\kappa}R + \mathcal{L}_{\mathrm{F}}\right)\sqrt{-g}\,\mathrm{d}^4x. \tag{15}$$

The equations of motion coming from the stationary-action principle now become

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \kappa T_{\mu\nu}, \tag{16}$$

where

$$T_{\mu\nu} = \frac{-2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\mathcal{L}_{\mathrm{F}})}{\delta g^{\mu\nu}} = -2\frac{\delta\mathcal{L}_{\mathrm{F}}}{\delta g^{\mu\nu}} + g_{\mu\nu}\mathcal{L}_{\mathrm{F}}, \tag{17}$$

is the stress-energy tensor and $\kappa = 8\pi G/c^4$ has been chosen such that the non-relativistic limit yields the usual form of Newton's gravity law.

3. Path Integral Formulation of Statistical Gravity

[. . .] e il suon di lei. Così tra questa
immensità s'annega il pensier mio:
e il naufragar m'è dolce in questo mare.
Giacomo Leopardi "L' Infinito"

Then, the action for Einstein's theory of general relativity is one for a particular field theory where the field is the metric tensor $g_{\mu\nu}(x)$, a symmetric tensor with 10 independent components, each of which is a smooth function of four variables. We will indicate all these components with the notation $\{g\}(x)$. We will also work in euclidean time $x^0 \equiv ct \to ict$ so that the metric signature becomes $(++++)$.

The thermal average of an observable $\mathcal{O}[\{g\}(x)]$ will then be given by the following expression:

$$\langle\mathcal{O}\rangle = \frac{\int \mathcal{O}[\{g\}(x)]\exp(-vS)\,\mathcal{D}^{10}\{g\}(x)}{\int \exp(-vS)\,\mathcal{D}^{10}\{g\}(x)}, \tag{18}$$

so that $\langle 1 \rangle = 1$. Here $1/v$ is a positive constant of the dimension of energy times the length, $ct \in [0, \beta]$ where $\beta = 1/\tilde{k}_B\tilde{T}$, $\tilde{k}_B$ is a Boltzmann constant of dimensions of one divided by the length and by degree Kelvin, and $\tilde{T}$, an *effective* temperature in degrees Kelvin. Since the thermal average involves taking a trace, we must have $g_{\mu\nu}(t + \beta/c, \mathbf{x}) = g_{\mu\nu}(t, \mathbf{x})$, where we denote with $\mathbf{x} \equiv (x^1, x^2, x^3)$ a spatial point. S is the Einstein–Hilbert action of Equation (15). We will also require periodic spatial boundary conditions on the finite volume $\Omega \subset \mathbb{R}^3$, which is the closest thing to a formal thermodynamic limit. As usual, we will use $\mathcal{D}^{10}\{g\}(x) \equiv \prod_x d^{10}\{g\}(x)$ and the functional integrals will be calculated on a lattice using the path integral Monte Carlo (PIMC) method [2]. Moreover we will choose $d^{10}\{g\}(x) \equiv \prod_{\mu\leq\nu} dg^{\mu\nu}(x)$, where the 10-dimensional space of the 10 independent components of the symmetric metric tensor is assumed to be flat.

The real metric fields $\{g\}$ take the values $\{g\}(x)$ on each site of a periodic 4-dimensional lattice of lattice spacing a, the ultraviolet cutoff, spatial periodicity $L = Na$, and temporal periodicity $\beta = N_0 a$. The metric path is a closed loop on a 4-dimensional surface of a 5-dimensional β-cylinder. We are interested in reaching the continuum limit by taking $L = Na$ fixed and letting $N \to \infty$ at fixed volume L^3. As for the temporal periodicity we can initially treat it similarly to spatial periodicity, taking $\beta = L$ and $N_0 = N$. On the other hand, we can distinguish the two periodicities allowing the effective temperature $\tilde{T} = 1/\tilde{k}_B\beta$ to vary so that the number of discretization points for the imaginary time interval $[0, \beta]$ will be $N_0 = \beta/a$. The PIMC simulation (the computer experiment) may use the Metropolis algorithm [8,9] to calculate the discretized version of Equation (18), which is a $10N^3N_0$ multidimensional integral. So, clearly, the computational resources needed to perform a given simulation depend critically on a. We may thus use natural Planck units $c = \hbar = k_B = 1$. The simulation is started from any initial condition. One MC step consists of the random displacement of each one of the $10N^3N_0$ independent components of $\{g\}$ as follows:

$$g^{\mu\nu} \to g^{\mu\nu} + (2\eta - 1)\delta, \tag{19}$$

where η is a uniform pseudo random number in $[0, 1]$ and δ is the amplitude of the displacement. Each one of these $10N^3N_0$ moves is accepted if $\exp(-v\Delta S) > \eta$, where ΔS is the change in the action due to the move and is rejected otherwise. The amplitude δ is chosen in such a way so as to have acceptance ratios as close to $1/2$ as possible, and is kept constant during the evolution of the simulation. One simulation consists of a large number of p steps. The statistical error on the average $\langle \mathcal{O} \rangle$ will then depend on the correlation time, $\tau_{\mathcal{O}}$, necessary to decorrelate the property $\mathcal{O}$ which will be determined as $\sqrt{\sigma^2_{\mathcal{O}}\tau_{\mathcal{O}}/[p10N^3N_0]}$, where $\sigma^2_{\mathcal{O}}$ is the intrinsic variance for $\mathcal{O}$. Our estimate of the path integrals will be generally subject to three sources of numerical uncertainties as follows: one due to the *statistical error*, one due to the *spacetime discretization*, and one due to the *finite-size effects*. Of these, the statistical error scales like $P^{-1/2}$, where $P = p10N^3N_0$ is the computer time, the discretization of spacetime is responsible for the distance from the continuum limit (which corresponds to a lattice spacing $a \to 0$), and the finite-size effects stem from the necessity to approximate the infinite spacetime system with one in a hypertorus of volume $L^3\beta$.

This makes sense since the Einstein theory does not predict a curvature of spacetime due to temperature, so it does not consider the virial theorem of statistical physics. According to the virial theorem, the temperature of a portion of spacetime should be related to kinetic energy which in turn should enter the stress energy tensor and be responsible for the curvature of spacetime according to Equation (16). Here, we are thinking of $\mathcal{L}_F$ as a 'potential energy' density of interaction of the metric field. Therefore we should take care of the effect of temperature in some other way. Here, we propose using the usual formalism of statistical physics to estimate the thermal averages $\langle \ldots \rangle$ of observables that depend on the metric tensor like, for example, the Riemann curvature tensor or the Christoffel symbols. This requires us to use a path integral over a 10-dimensional space like in Equation (18).

Then, both the curvature of spacetime and the geodesic equation of motion of a point particle will be influenced by temperature and one should replace them with their thermally averaged versions more correctly. In the classical high temperature limit, applying a time average to the contracted Einstein field Equation (16), $-R = \kappa T^{\mu}_{\mu}$, assuming $\mathcal{L}_F$ is independent of $g^{\mu\nu}$ so that $T^{\mu}_{\mu} = 4\mathcal{L}_F$, and replacing the time average with the ensemble average of Equation (18) in the first member, we soon reach (see the Appendix A) the following relation: $-2\kappa/\beta\bar{v} = \frac{\kappa}{2}\langle T^{\mu}_{\mu}\rangle_t$, with $\langle \ldots \rangle_t = \lim_{\tau\to\infty}\int_0^{\tau}\ldots dt/\tau$ as the time average, $\bar{v}$ as another constant of dimension length squared divided by energy, κ having the dimensions of length divided by energy, and the stress energy tensor elements having the dimensions of energy density. This gives a temperature definition of $-\tilde{k}_B\tilde{T} = \frac{\bar{v}}{4}\langle T^{\mu}_{\mu}\rangle_t$.

In Equation (18), we assumed a constant temperature throughout the whole accessible spacetime. This can be a too restrictive condition and it might be necessary to think about a temperature scalar field $\tilde{T}(x)$ which gives the value of the temperature in a neighborhood of a given event x. Actually, we are bound to choose $\tilde{T}(\mathbf{x})$ as the temperature in a neighborhood of a given spatial point $\mathbf{x}$ since we must require $x^0 \in [0, \beta(\mathbf{x})]$. Furthermore, the temperature field $\tilde{T}(\mathbf{x}) = 1/\tilde{k}_B\beta(\mathbf{x})$ could be available experimentally.

In Equation (18), we assumed that what gives rise to the thermal average is the product of the 10 independent components of the tensor path integral $\mathcal{I}^{\mu\nu} = \int \ldots \mathcal{D}g^{\mu\nu}(x)$. This may be regarded as a rather arbitrary finding but it gives us something that is numerically amenable and directly accessible through the Monte Carlo path integral. Moreover it gives the correct classical high temperature limit $\langle \mathcal{O}\rangle \to \langle \mathcal{O}_c\rangle$, where $\mathcal{O}_c$ is the value of the observable temperature at the metric of the solution of the classical Einstein field Equation (16).

In order to make these arguments precise, and to treat the integral in Equation (18) correctly, we immediately recognize that it is necessary to split the time component from the three spatial components in line with the Arnowitt–Deser–Misner (ADM) foliation formalism. In this respect, affine quantization, as explained in Ref. [1], may become useful.

4. Conclusions

We propose a way to include into Einstein's general relativity theory the effect of temperature. This involves the use of a path integral on a ten-dimensional flat space in a four-dimensional spacetime lattice hypertorus. It may prove necessary to introduce a temperature scalar field function of the position within the spatial volume under examination. From the definition of the thermal average, we will have that at large temperatures, i.e., small β, the quantum effects of statistics on the spacetime fabric will be irrelevant. On the other hand, the quantum effects will become important at low temperatures, i.e., large β. This means that the quantum effects are negligible whenever the path in the metric components extends over a short imaginary time interval, i.e., the hypertorus 'radius' around the time curled dimension is small and reduces to something like a round closed hyper'string' whose thickness varies along its length. We will call our theory the FEBB after the initials of the surnames of the key contributors of the three main advances in theoretical physics as follows: Albert Einstein for physics of gravitation, Ludwig Boltzmann for statistical physics, and Niels Bohr for quantum physics.

The internal consistency of our FEBB theory requires a definition of 'temperature' as

$$\tilde{T} = -\frac{\bar{v}}{4\tilde{k}_B}\langle T^{\mu}_{\mu}\rangle_t, \tag{20}$$

where $\tilde{k}_B$ is a Boltzmann constant with dimensions of one divided by the length and degrees Kelvin.

Affine quantization, as explained in Ref. [1], may be useful in making these arguments precise and for the physical meaning of the factor $\bar{v}$.

We conclude by noting that for an ideal fluid in thermodynamic equilibrium in imaginary time $T^\mu_\mu = 5P + \rho c^2$, where P is the pressure and ρ the mass-energy density, the non-relativistic and non-quantum limits required are as follows:

$$-4\frac{\tilde{k}_B\tilde{T}}{\bar{v}} - \rho c^2 \to 5nk_BT, \tag{21}$$

where k_B and T are the usual Boltzmann constant and absolute temperature, respectively. Thus, we recover the usual ideal gas equation of state $P = nk_BT$, where $P = \langle P\rangle_t = \langle P\rangle$ is the pressure and $n = \langle n\rangle_t = \langle n\rangle$ is the number density. Note that in Equation (21) $\bar{v} > 0$, $\rho c^2 > 0$, and the right hand side is also a positive quantity. However, thanks to the circular boundary conditions around the imaginary time, we are free to choose $\beta < 0$. Furthermore, this should be the case (Note that $\int_0^\infty e^{-\alpha y}\, dy = \frac{1}{\alpha}$ when $\alpha > 0$. So that $[(d/d\alpha)\int_0^\infty e^{-\alpha y}\, dy]/\int_0^\infty e^{-\alpha y}\, dy = -\frac{1}{\alpha}$. Moreover in imaginary time we have a negative Ricci scalar curvature, i.e., $R < 0$, so that we must have $\beta < 0$) since by tracing the Einstein field equations we follow, in imaginary time, $-R = \kappa T^\mu_\mu > 0$, i.e., $R < 0$. Since in the non-relativistic limit, $c \to \infty$, we have in imaginary time $\rho c^2 \to nm_0c^2 - n\frac{1}{2}m_0v^2$, with m_0 as the rest mass of the particles and v as their velocities. We will find the following for the equipartition theorem $\langle \rho c^2\rangle \to nm_0c^2 - \frac{3}{2}k_BT$ we require, in the non-relativistic ($c \to \infty$) non-quantum ($\tilde{T} \to \infty$) limits:

$$4\frac{\tilde{k}_B\tilde{T}}{\tilde{v}} - nm_0c^2 \to \frac{7}{2}nk_BT, \tag{22}$$

where we called $\tilde{v} = -\bar{v}$. We then see that $\tilde{T}$ should be chosen to be proportional to the number density n. This is a necessary constraint if we are to maintain our initial belief that a Wick rotation will take us from quantum gravity to statistical gravity. Our Equation (22) makes sense, since infinity minus infinity is an indeterminate form and can be equal to a finite result.

We ask the reader, could our effective (obscure) temperature give some hints in the search for the **dark matter** in the universe. Also, could our new theory explain the recent findings about the **tension** in the universe [10].

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Appendix A. Derivation of the Virial Theorem

Taking the thermal average of the trace of the Einstein field Equation (16), we find

$$\langle -R\rangle = \kappa\langle T^\mu_\mu\rangle. \tag{A1}$$

Now we notice that we can evaluate the left hand side as follows:

$$\langle -R\rangle \quad = \quad \frac{\int(-R)\exp(-vS)\,\mathcal{D}^{10}\{g\}(x)}{\int \exp(-vS)\,\mathcal{D}^{10}\{g\}(x)}. \tag{A2}$$

In the high temperature limit we will have, from Equation (15),

$$vS \approx \beta\bar{v}\left(\frac{1}{2\kappa}R + \mathcal{L}_\mathrm{F}\right). \tag{A3}$$

Note that this is only an approximation and we would more correctly need a 3 + 1 splitting of space from imaginary time. If we do not account for these details, for the time being, we reach the following result:

$$\langle -R\rangle \approx \frac{2\kappa}{\bar{v}}\frac{(d/d\beta)\int \exp(-vS)\,\mathcal{D}^{10}\{g\}(x)}{\int \exp(-vS)\,\mathcal{D}^{10}\{g\}(x)} + 2\kappa\langle\mathcal{L}_F\rangle. \tag{A4}$$

Now, assuming $\mathcal{L}_F$ is independent from the metric tensor, we find from Equation (17) that $T^\mu_\mu = 4\mathcal{L}_F$ and also

$$\langle -R\rangle \approx \frac{2\kappa}{\bar{v}}\frac{(d/d\beta)\int \exp(-vS)\,\mathcal{D}^{10}\{g\}(x)}{\int \exp(-vS)\,\mathcal{D}^{10}\{g\}(x)} + \frac{\kappa}{2}\langle T^\mu_\mu\rangle \tag{A5}$$

$$= -\frac{2\kappa}{\beta\bar{v}} + \frac{\kappa}{2}\langle T^\mu_\mu\rangle, \tag{A6}$$

where in the last equality, we took profit of the fact that $\mathcal{L}_F$ is independent from the metric tensor components $\{g\}$, so that its contribution simplifies from the two path integrals in the numerator and in the denominator (Note that $\int_0^\infty e^{-\alpha y}\,dy = \frac{1}{\alpha}$ when $\alpha > 0$. So that $[(d/d\alpha)\int_0^\infty e^{-\alpha y}\,dy]/\int_0^\infty e^{-\alpha y}\,dy = -\frac{1}{\alpha}$. Moreover, in imaginary time, we have a negative Ricci scalar curvature, i.e., $R < 0$, so that we must have $\beta < 0$).

Incorporating this result into Equation (A1), we reach the following result:

$$-\frac{2\kappa}{\beta\bar{v}} + \frac{\kappa}{2}\langle T^\mu_\mu\rangle \approx \kappa\langle T^\mu_\mu\rangle, \tag{A7}$$

or

$$-\frac{2\kappa}{\beta\bar{v}} \approx \frac{\kappa}{2}\langle T^\mu_\mu\rangle. \tag{A8}$$

which gives the desired result upon replacing the thermal average with the time average introduced in the main text $\langle\ldots\rangle \to \langle\ldots\rangle_t$.

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