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# The secret to fixing incorrect canonical quantizations

John R. Klauder[1], Riccardo Fantoni[2,*]

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## Abstract

Covariant scalar field quantizations will be called $(\varphi^r)_n$, where $r$ denotes the power of the interaction term and $n = s + 1$, where $s$ is the spatial dimension and 1 adds time. Models where $r < 2n/(n-2)$ can be treated by canonical quantization, while models where $r > 2n/(n-2)$ are trivial or, if treated as a unit, emerge as 'free theories'. Moreover, according to canonical quantization, models where $r = 2n/(n-2)$, e.g., $r = n = 4$, also become 'free theories', which must be considered quantum failures. However, there exists a different approach called affine quantization. This approach promotes a different set of classical variables to become the basic quantum operators and offers different results. It is well known that the canonical quantization of $\varphi^4_4$ fails. This article addresses this failure alongside solutions to other problems.

**Keywords:** *canonical quantization, affine quantization, quantum field theory*

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## 1. Introduction

There are many models with the same 'illness' as that of $\varphi^4_4$ [1–12]. The secret to a valid canonical quantization (CQ) is remarkably simple. All you need is the addition of a single, fixed potential, which is not seen in the classical Hamiltonian, but which puts terms in proper position elsewhere. This single, additional, potential is $2\hbar^2/\varphi(x)^2$. This potential can be added to the models for $\varphi^4_4$ and $\varphi^8_4$. The additional potential, put just after $\hat{\pi}(x)^2$, is all that is needed.

## 2. Results when removing $\varphi(x) = 0$

The special $\hbar$-term has arisen from the fact that $\varphi(x) = 0$ has been removed, which means that the momentum is no longer self-adjoint. The next step introduces $\hat{\kappa}(x) = [\hat{\pi}(x)^\dagger\varphi(x)+\varphi(x)\hat{\pi}(x)]/2$, and with scaling, it becomes

$$\begin{aligned} \pi(x)^2 &= \kappa(x)^2/\varphi(x)^2 \to \hat{\kappa}(x)\varphi(x)^{-2}\hat{\kappa}(x) \\ &\Rightarrow \hat{\pi}(x)^2 + b\hbar^2/\varphi(x)^2, \end{aligned} \tag{1}$$

where the factor $b = 2$ has been chosen to fix this particular problem. Observe that choosing $\varphi(x) \neq 0$ has permitted the introduction of the 'polynomial'-like term $2\hbar^2/\varphi(x)^2$.

### 2.1. How this scaling functions

Initially, $\hat{\kappa}(x)\varphi(x)^{-2}\hat{\kappa}(x) = \hat{\pi}^2 + \hbar^2\delta(0)^{2s}/\varphi(x)^2$, where $\delta(0)$ is Dirac's special function. Where $\delta(x) = 0$ for all $0 < |x|$, while $\int\delta(x)\ dx = 1$ leads to $\delta(0) = \infty$. Now our $\infty$ is reduced to $b\hbar^2W^2 < \infty$, and $W$ will be set to $\infty$ later on, where $W$ is a scaling factor.

This now becomes $\hat{\pi}(x)^2+b\hbar^2W^2/\varphi(x)^2$. Next, $(\hat{\pi}(x)^2\ \&\ \varphi(x)^2) \to W(\hat{\pi}(x)^2\ \&\ \varphi(x)^2)$. This leads to $W\hat{\pi}(x)^2 + b\hbar^2W^2/W\varphi(x)^2$, and now a full multiplication by $W^{-1}$ leads to the final result which is $\hat{\pi}(x)^2 + b\hbar^2/\varphi(x)^2$. Now $W$ can be set to $\infty$.

## 3. Selected topics of affine quantization

A major feature of CQ is that $-\infty < q\ \&\ \varphi(x) < \infty$ either in quantum mechanics where $q$ is position or in scalar field theory where $\varphi(x)$ is the field. It is this fact which affine quantization (AQ) overcomes by introducing a variety of parts of incomplete space, such as these retained spaces, $q > 0,\ |q| > 0,\ q^2 < b^2,\ q^2 > b^2$, etc. For quantum field theory, the most important change is that $\varphi(x) \neq 0$ and that equation has been fully removed.

Note that CQ requires $0 \leq |\varphi(x)| < \infty$, while AQ seeks to find missing equations, which show that a specific field value, namely, $\varphi(x) = 0$ is removed [13]; in other words, when $0 < |\varphi(x)| < \infty$.

### 3.1. An introduction to affine quantization (AQ)

Only AQ can correctly solve all examples that have missing space regions and can do so correctly simply with the remaining space examples.

There is something else that CQ fails on, namely the "Particle in a Box" example. This example is with missing space and has been traditionally 'solved' using CQ. However, that very model can, and has, been correctly solved now by using AQ [7].

If you wish to read up on AQ, there are two examples given in [7, 14], where AQ has been well explained.

[1]Department of Physics and Department of Mathematics, University of Florida, Gainesville, FL 32611, USA.
[2]Department of Physics, University of Trieste, 34151 Grignano, Trieste, Italy.
*email: riccardo.fantoni@scuola.istruzione.it

## 4. Conclusions

There have been many models with the same problems as that of $\varphi_4^4$ [1–7]. The secret to a valid canonical quantization is remarkably simple. What is needed is the addition of a single, fixed potential, which is not seen in other texts, but which puts terms in proper position elsewhere. This single, additional, potential is $2\hbar^2/\varphi(x)^2$, alongside $\hat{\pi}(x)^2$. It is noteworthy that this potential forces $0 < |\varphi(x)| < \infty$, which leads to $0 < |\varphi(x)|^r < \infty$ and guarantees that almost all other potentials remain finite.

That additional potential is all you will need.

This solution is possible due to affine quantization [14, 15].

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## Conflict of interest

The authors declare no conflict of interest.

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Data supporting these findings are available upon reasonable requests to the authors.

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## References

1. Aizenman M. Proof of the triviality of $\phi_d^4$ field theory and some mean-field features of Ising models for $d > 4$. Phys Rev Lett. 1981;47: 886(E). doi: 10.1103/PhysRevLett.47.1
2. Fröhlich J.On the triviality of $\lambda\phi_d^4$ theories and the approach to the critical point in $d \geq 4$ dimensions. Nucl Phys B. 1982;200: 281. doi: 10.1016/0550-3213(82)90088-8
3. Freedman B, Smolensky P, Weingarten D. Monte Carlo evaluation of the continuum limit of $\phi_4^4$ and $\phi_3^4$. Phys Lett. 1982;113B: 481. doi: 10.1016/0370-2693(82)90790-0
4. Siefert J, Wolff U.Triviality of $\varphi^4$ theory in a finite volume scheme adapted to the broken phase. Phys Lett B. 2014;733:11. doi: 10.1016/j.physletb.2014.04.013
5. Wolff U.Triviality of four dimensional $\phi^4$ theory on the lattice. Scholarpedia. 2014;9: 7367. doi: 10.4249/scholarpedia.7367
6. Aizenman M, Duminil-Copin H.Marginal triviality of the scaling limits of critical 4D Ising and $\phi_4^4$ models. Ann Math. 2021;194:163. doi: 10.4007/annals.2021.194.1.3
7. Klauder JR, Fantoni R. The magnificent realm of affine quantization: valid results for particles, fields, and gravity. Axioms. 2023;12: 911. doi: 10.3390/axioms12100911
8. Itzykson C, Zuber J-B. Quantum field theory. New York: McGraw-Hill; 1980.
9. Callaway DJE. Triviality pursuit: can elementary scalar particles exist? Phys Rep. 1988;167(5):241–320. doi: 10.1016/0370-1573(88)90008-7
10. Simon B. Quadratic forms and Klauder's phenomenon: a remark on very singular perturbations. J Funct Anal. 1973;14:295. doi: 10.1016/0022-1236(73)90074-8
11. Avossevon GYH, Hounguevou JV, Takay DS. Conventional and enhanced canonical quantization, application to some simple manifolds. J Mod Phys. 2013;4:1476–85. doi: 10.4236/jmp.2013.411177

12. Fanuel M, Zonetti S. Affine quantization and the initial cosmological singularity. EPL. 2013;101:10001; arXiv:1203.4936v3. doi: 10.1209/0295-5075/101/10001

13. Watch: "Why the number 0 was banned for 1,500 years". [cited 2024 Sep 26]. Available from: https://www.youtube.com/watch?v=ndmwB8F2kxA&ab_channel=UpandAtom

14. Klauder JR. The benefits of affine quantization. J High Energy Phys Gravit Cosmol. 2020;6: 175. doi: 10.4236/jhepgc.2020.62014

15. [cited 2024 16]. Available from: https://en.wikipedia.org/wiki/Affine_group