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WEAK POTENTIAL

$$\boxed{\left(\mathcal{E}^0_{\vec{k}-\vec{K}} - \mathcal{E}\right) c_{\vec{k}-\vec{K}} + \sum_{\vec{K}'} U_{\vec{K}'-\vec{K}}\, c_{\vec{k}-\vec{K}'} = 0} \quad (*)$$

FREE ELECTRONS: (Drop "→" over $\vec{k}$, ...)

$$U_{\vec{K}'-\vec{K}} = 0 \Rightarrow \underbrace{\left(\mathcal{E}^0_{\vec{k}-\vec{K}} - \mathcal{E}\right)}_{=0} \underbrace{c_{k-K}}_{=0 \text{ (NO, trivial)}} = 0$$

case (I) NON-DEGENERATE

for a given k, $\exists!\ K_1$ s.t.:

(1) $\mathcal{E}^0_{\vec{k}-\vec{K}_1} = \mathcal{E}$; $c_{k-K_1} \neq 0$

$\forall K' \neq K_1 \Rightarrow \mathcal{E}^0_{k-K'} \neq \mathcal{E}^0_{k-K_1}$

case (II) DEGENERATE

for a given k, $\exists$ a set $\{K_i\}$ s.t.

(2) $\mathcal{E}^0_{k-K_1} = \mathcal{E}^0_{k-K_2} = \dots$

$c^0_{k-K_1}, c^0_{k-K_2} \dots \neq 0$

WEAKLY PERTURBED CASE:

$U \neq 0 \Rightarrow c_{k-K} = c_{k-K}(U)$ NOT very $\neq$ from c^0_{k-K}

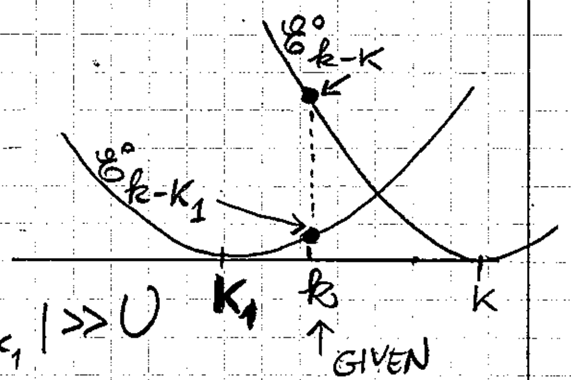
case (I) NON DEGENERATE

Suppose $\exists!\ K_1$ s.t. (1)

Therefore, $\forall K \neq K_1$:

$\mathcal{E}^0_{k-K} \neq \mathcal{E}^0_{k-K_1}$

and more precisely $|\mathcal{E}^0_{k-K} - \mathcal{E}^0_{k-K_1}| \gg U$



How to obtain an expression for (approximate) c_{k-K_1} ? and c_{k-K} $(K \neq K_1)$?

Hint (1) start from $(*)$,

a) specify for K_1 → impossible to find directly an expression for c_{k-K_1}

b) " for $K \neq K_1$, split $\sum_{K'} = \underset{\text{in } K_1}{\underline{\text{term}}} + \underbrace{\sum_{K \neq K_1}}_{\text{small}}$!

$\Rightarrow$ solve for c_{k-K} $(K \neq K_1)$ and obtain:

$$(**) \quad c^{(1)}_{k-K} = \frac{U_{K_1-K}\, c_{k-K_1}}{\mathcal{E} - \mathcal{E}^0_{k-K}} + \mathcal{O}(U^2)$$

(2) Put $(**)$ in a), after splitting also there

$$\sum_{K'} = \underbrace{\text{term}}_{\text{in } K_1} + \sum_{K \neq K_1}$$

$\hookrightarrow$ which gives $\emptyset$ since $U_{K_1-K_1} = 0$

(3) Obtain:

$$(\mathcal{E} - \mathcal{E}^0_{k-K_1})\, c_{k-K_1} = \sum_{K \neq K_1} U_{K-K_1} \frac{U_{K_1-K}\, c_{k-K_1}}{\mathcal{E} - \mathcal{E}^0_{k-K}} + \mathcal{O}(U^3)$$

(4) Divide by $c_{k-K_1} \neq 0$ and obtain

$$\boxed{\mathcal{E} = \mathcal{E}^0_{k-K_1} + \sum_K{}' \frac{|U_{K-K_1}|^2}{\mathcal{E}^0_{k-K_1} - \mathcal{E}^0_{k-K}} + \mathcal{O}(U^3)}$$

case (II) DEGENERATE

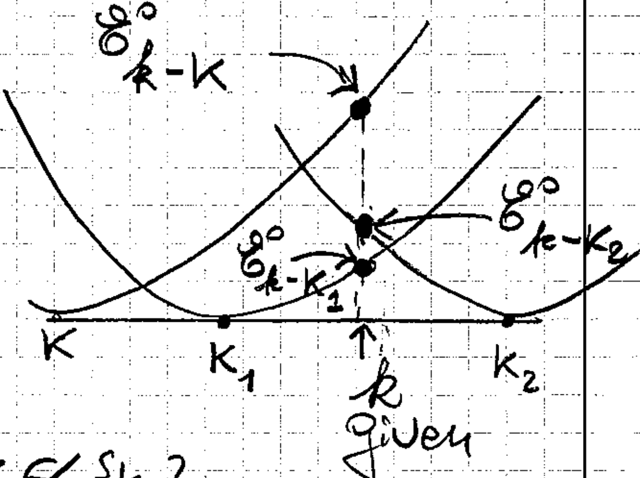
Suppose $\{K_1, K_2, \dots\}$

s.t. $|\mathcal{E}^0_{k-K_i} - \mathcal{E}^0_{k-K_j}| \approx \mathcal{O}(U)$

whereas

$|\mathcal{E}^0_{k-K} - \mathcal{E}^0_{k-K_i}| \gg U$

$\forall K \notin \{K_i\}$



Same procedure, but splitting $\sum_{K'}$ in $(*)$ as $\sum_{\{K_i\}} + \sum_{K \notin \{K_i\}}$ leaves a term $\mathcal{O}(U)$ in (a)

$$\Rightarrow \quad (\mathcal{E} - \mathcal{E}^0)\, c_{k-K_i} \cong \sum_{K_j} U_{K_j-K_i}\, c_{k-K_j}$$