An introduction to quantum spin liquids
Part II

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1. Mean-field approaches to spin liquids
   - Non-standard mean-field approaches to spin-liquid phases
   - Fermionic representation of a spin-1/2
   - Projective symmetry group (PSG)

2. Beyond the mean-field approaches
   - "Low-energy" gauge fluctuations
   - Variational Monte Carlo for fermions

3. Numerical results
   - An example: the Heisenberg model on the Kagome lattice

Standard mean-field approach

Consider the spin-1/2 Heisenberg model on a generic lattice

$$\mathcal{H} = \sum_{ij} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$$

In a standard mean-field approach, each spin couples to an effective field generated by the surrounding spins:

$$\mathcal{H}_{\mathrm{MF}} = \sum_{ij} J_{ij} \left\{ \langle \mathbf{S}_i \rangle \cdot \mathbf{S}_j + \mathbf{S}_i \cdot \langle \mathbf{S}_j \rangle - \langle \mathbf{S}_i \rangle \cdot \langle \mathbf{S}_j \rangle \right\}$$

However, by definition, spin liquids have a zero magnetization:

$$\langle \mathbf{S}_i \rangle = 0$$

How can we construct a mean-field approach for such disordered states?

We need to construct a theory in which all classical order parameters are vanishing

Halving the spin operator

- The first step is to decompose the spin operator in terms of spin-1/2 quasi-particles creation and annihilation operators.
- One possibility is to write:

$$S_i^\mu = \frac{1}{2} c_{i,\alpha}^\dagger \sigma_{\alpha,\beta}^\mu c_{i,\beta}$$

$\sigma_{\alpha,\beta}^\mu$ are the Pauli matrices

$$\sigma^x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma^y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$c_{i,\alpha}^\dagger$ ($c_{i,\beta}$) creates (destroys) a quasi-particle with spin-1/2
These may have various statistics, e.g., bosonic or fermionic

At this stage, splitting the original spin operator in two pieces is just a formal trick.
Whether or not these quasi-particles are true elementary excitations is THE question

Fermionic representation of a spin-1/2

- A faithful representation of spin-1/2 is given by:

$$
\begin{aligned}
S_i^z &= \frac{1}{2}\left(c^\dagger_{i,\uparrow}c_{i,\uparrow} - c^\dagger_{i,\downarrow}c_{i,\downarrow}\right)\\
S_i^+ &= c^\dagger_{i,\uparrow}c_{i,\downarrow}\\
S_i^- &= c^\dagger_{i,\downarrow}c_{i,\uparrow}
\end{aligned}
$$

$\{c_{i,\alpha}, c^\dagger_{j,\beta}\} = \delta_{ij}\delta_{\alpha\beta}$
$\{c_{i,\alpha}, c_{j,\beta}\} = 0$
$c^\dagger_{i,\uparrow}$ (or $c^\dagger_{i,\downarrow}$) changes S_i^z by $1/2$ (or $-1/2$) and creates a "spinon"

- For a model with one spin per site, we must impose the constraints:

$$\boxed{c^\dagger_{i,\uparrow}c_{i,\uparrow} + c^\dagger_{i,\downarrow}c_{i,\downarrow} = 1} \qquad \boxed{c_{i,\uparrow}c_{i,\downarrow} = 0}$$

- Compact notation by using a 2×2 matrix:

$$\Psi_i = \begin{bmatrix} c_{i,\uparrow} & c^\dagger_{i,\downarrow} \\ c_{i,\downarrow} & -c^\dagger_{i,\uparrow} \end{bmatrix} \qquad S_i^\mu = -\frac{1}{4}\mathrm{Tr}\left[\sigma^\mu \Psi_i \Psi_i^\dagger\right] \qquad G_i^\mu = \frac{1}{4}\mathrm{Tr}\left[\sigma^\mu \Psi_i^\dagger \Psi_i\right] = 0$$

Local redundancy and "gauge" transformations

$$S_i^\mu = -\frac{1}{4}\mathrm{Tr}\left[\sigma^\mu \Psi_i \Psi_i^\dagger\right]$$

$$\mathbf{S}_i \cdot \mathbf{S}_j = \frac{1}{16}\sum_\mu \mathrm{Tr}\left[\sigma^\mu \Psi_i \Psi_i^\dagger\right] \mathrm{Tr}\left[\sigma^\mu \Psi_j \Psi_j^\dagger\right] = \frac{1}{8}\mathrm{Tr}\left[\Psi_i \Psi_i^\dagger \Psi_j \Psi_j^\dagger\right]$$

• Spin rotations are left rotations:

$$\Psi_i \to R_i \Psi_i$$

The Heisenberg Hamiltonian is invariant under global rotations

• The spin operator is invariant upon local SU(2) "gauge" transformations, right rotations:

$$\Psi_i \to \Psi_i W_i$$

$$\mathbf{S}_i \to \mathbf{S}_i$$

There is a huge redundancy in this representation

Affleck, Zou, Hsu, and Anderson, Phys. Rev. B **38**, 745 (1988)

Mean-field approximation

- We transformed a spin model into a model of interacting fermions (subject to the constraint of one-fermion per site)
- The first approximation to treat this problem is to consider a mean-field decoupling:

$$\Psi_i^\dagger \Psi_j \Psi_j^\dagger \Psi_i \to \langle \Psi_i^\dagger \Psi_j \rangle \Psi_j^\dagger \Psi_i + \Psi_i^\dagger \Psi_j \langle \Psi_j^\dagger \Psi_i \rangle - \langle \Psi_i^\dagger \Psi_j \rangle \langle \Psi_j^\dagger \Psi_i \rangle$$

We define the mean-field 2×2 matrix

$$U_{ij}^0 = \frac{J_{ij}}{4} \langle \Psi_i^\dagger \Psi_j \rangle = \frac{J_{ij}}{4} \begin{bmatrix} \langle c_{i,\uparrow}^\dagger c_{j,\uparrow} + c_{i,\downarrow}^\dagger c_{j,\downarrow} \rangle & \langle c_{i,\uparrow}^\dagger c_{j,\downarrow}^\dagger + c_{j,\uparrow}^\dagger c_{i,\downarrow}^\dagger \rangle \\ \langle c_{i,\downarrow} c_{j,\uparrow} + c_{j,\downarrow} c_{i,\uparrow} \rangle & -\langle c_{j,\downarrow}^\dagger c_{i,\downarrow} + c_{j,\uparrow}^\dagger c_{i,\downarrow} \rangle \end{bmatrix} = \begin{bmatrix} \chi_{ij} & \eta_{ij}^* \\ \eta_{ij} & -\chi_{ij}^* \end{bmatrix}$$

- $\chi_{ij} = \chi_{ji}^*$ is the spinon hopping
- $\eta_{ij} = \eta_{ji}$ is the spinon pairing

Mean-field approximation

The mean-field Hamiltonian has a BCS-like form:

$$\mathcal{H}_{MF} = \sum_{ij} \chi_{ij}(c^\dagger_{j,\uparrow} c_{i,\uparrow} + c^\dagger_{j,\downarrow} c_{i,\downarrow}) + \eta_{ij}(c^\dagger_{j,\uparrow} c^\dagger_{i,\downarrow} + c^\dagger_{i,\uparrow} c^\dagger_{j,\downarrow}) + h.c.$$

$$+ \sum_i \mu_i \,(c^\dagger_{i,\uparrow} c_{i,\uparrow} + c^\dagger_{i,\downarrow} c_{i,\downarrow} - 1) + \sum_i \zeta_i \, c^\dagger_{i,\uparrow} c^\dagger_{i,\downarrow} + h.c.$$

- $\{\chi_{ij}, \eta_{ij}, \mu_i\,, \zeta_i\,\}$ define the mean-field Ansatz

At the mean-field level χ_{ij} and η_{ij} are fixed numbers

The SU(2) gauge is broken!

It is restored when computing quantities in sub-space with one electron per site (the physical Hilbert space)

Elitzur, Phys. Rev. D. **12** 3978 (1975)

Mean-field approximation and gauge symmetry

- Let $|\Phi_{MF}(U^0_{ij})\rangle$ be the ground state of the mean-field Hamiltonian (with a given Ansatz for the mean-field U^0_{ij})

- Let us consider an arbitrary *site-dependent* SU(2) matrix W_i (gauge transformation)

$$\Psi_i \to \Psi_i W_i$$

Leaves the spin unchanged $\mathbf{S}_i \to \mathbf{S}_i$.

$$U^0_{ij} \to W_i \; U^0_{ij} W_j^\dagger$$

- Therefore, U^0_{ij} and $W_i \; U^0_{ij} W_j^\dagger$ define the same physical state (the same physical state can be represented by many different Ansätze U^0_{ij})

$$\langle 0| \prod_i c_{i,\alpha_i} |\Phi_{MF}(U^0_{ij})\rangle = \langle 0| \prod_i c_{i,\alpha_i} |\Phi_{MF}(W_i \; U^0_{ij} W_j^\dagger)\rangle$$

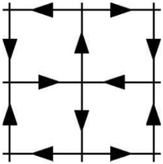
An example on the square lattice

- The staggered flux state is defined by

Affleck and Marston, Phys. Rev. B **37**, 3774 (1988)

$$j \in A \begin{cases} \chi_{j,j+x} = e^{i\Phi_0/4} \\ \chi_{j,j+y} = e^{-i\Phi_0/4} \end{cases}$$

$$j \in B \begin{cases} \chi_{j,j+x} = e^{-i\Phi_0/4} \\ \chi_{j,j+y} = e^{i\Phi_0/4} \end{cases}$$



- The d-wave "superconductor" state is defined by

Baskaran, Zou, and Anderson, Solid State Commun. **63**, 973 (1987)

$$\begin{cases} \chi_{j,j+x} = 1 \\ \chi_{j,j+y} = 1 \\ \eta_{j,j+x} = \Delta \\ \eta_{j,j+y} = -\Delta \end{cases}$$

- For $\Delta = \tan(\Phi_0/4)$, these two mean-field states become the same state after projection
- The mean-field spectrum is the same for the two states (it is invariant under SU(2) transformations)

- Ansätze that differ by a gauge transformation describe the same physical state
- A non-fully-symmetric mean-field Ansatz U^0_{ij} (e.g., breaking translational symmetry) may correspond to a fully-symmetric physical state

  Let us define a generic lattice symmetry (translations, rotations, reflections) by T:

$$TU^0_{ij} = U^0_{T(i)T(j)} \neq U^0_{ij}$$

  but still the physical state may have all lattice symmetries.
  Indeed, we can still play with gauge transformations.

- To have a fully-symmetric physical state, a gauge transformation G_i must exist, such that

$$\boxed{G_i^\dagger TU^0_{ij} G_j = G_i^\dagger U^0_{T(i)T(j)} G_j \equiv U^0_{ij}}$$

$\{T, G\}$ define the PSG:
for each lattice symmetry T, there is an associated gauge symmetry G

Wen's conjecture on quantum order

- In general, the PSG is not trivial
  (the set of gauge transformations G associated to lattice symmetries T is non-trivial)
- Distinct spin liquids have the same lattice symmetries (i.e., they are totally symmetric), but different PSGs (i.e., different gauge transformations G)
- Wen proposed to use the PSG to characterize quantum order in spin liquids
- As in the Landau's theory for classical orders, where symmetries define various phases, the PSG can be used to classify spin liquids
  (the PSG of an Ansatz is a universal property of the Ansatz)

Although Ansätze for different spin liquids have the same symmetry, the Ansätze are invariant under different PSG. Namely different sets of gauge transformations associated to lattice symmetries

Wen, Phys. Rev. B **65**, 165113 (2002)

"Low-energy" gauge fluctuations

- The SU(2) gauge structure

$$\Psi_i \to \Psi_i W_i$$

  is a "high-energy" gauge structure that only depends upon our choice on how to represent the spin operator [e.g., for the bosonic representation, it is U(1)]

- What are the "relevant" gauge fluctuations above a given mean-field Ansatz U_{ij}^0?

- The "relevant" ("low-energy") gauge fluctuations are determined by the IGG (Wen's conjecture)

  The IGG of a mean-field Ansatz is defined by all gauge symmetries that leave U_{ij}^0 unchanged:

$$\mathcal{G}_i U_{ij}^0 \mathcal{G}_j^\dagger = U_{ij}^0$$

  These are the "unbroken" gauge symmetries of the mean-field Ansatz

The PSG + IGG allow us to classify spin liquid phases

- Consider the square lattice and a Z_2 IGG, e.g. $\mathcal{G}_i = \pm\mathbb{I}$
- Consider the case where only translations T_x and T_y are considered

  Only two Z_2 spin liquids are possible:

$$
\begin{cases}
G_i(T_x) = \mathbb{I} & G_i(T_y) = \mathbb{I} \quad \rightarrow \quad U^0_{i,i+m} = U^0_m \\
G_i(T_x) = (-1)^{i_y}\mathbb{I} & G_i(T_y) = \mathbb{I} \quad \rightarrow \quad U^0_{i,i+m} = (-1)^{m_y i_x} U^0_m
\end{cases}
$$

- The case with also point-group and time-reversal symmetries is much more complicated

  Two classes of Z_2 spin liquids are possible:

$$
\begin{aligned}
G_i(T_x) &= \mathbb{I} & G_i(T_y) &= \mathbb{I} \\
G_i(P_x) &= \epsilon_{xpx}^{i_x}\epsilon_{xpy}^{i_y} g_{P_x} & G_i(P_y) &= \epsilon_{xpy}^{i_x}\epsilon_{xpx}^{i_y} g_{P_y} \\
G_i(P_{xy}) &= g_{P_{xy}} & G_i(T) &= \epsilon_t^{i} g_T
\end{aligned}
$$

$$
\begin{aligned}
G_i(T_x) &= (-1)^{i_y}\mathbb{I} & G_i(T_y) &= \mathbb{I} \\
G_i(P_x) &= \epsilon_{xpx}^{i_x}\epsilon_{xpy}^{i_y} g_{P_x} & G_i(P_y) &= \epsilon_{xpy}^{i_x}\epsilon_{xpx}^{i_y} g_{P_y} \\
G_i(P_{xy}) &= (-1)^{i_x i_y} g_{P_{xy}} & G_i(T) &= \epsilon_t^{i} g_T
\end{aligned}
$$

$$g_{Pxy}=\tau^0, \quad g_{P_x}=\tau^0, \quad g_{P_y}=\tau^0, \quad g_T=\tau^0; \qquad (67)$$
$$g_{Pxy}=\tau^0, \quad g_{P_x}=i\tau^3, \quad g_{P_y}=i\tau^3, \quad g_T=\tau^0; \qquad (68)$$
$$g_{Pxy}=i\tau^3, \quad g_{P_x}=\tau^0, \quad g_{P_y}=\tau^0, \quad g_T=\tau^0; \qquad (69)$$
$$g_{Pxy}=i\tau^3, \quad g_{P_x}=i\tau^3, \quad g_{P_y}=\tau^3, \quad g_T=\tau^0; \qquad (70)$$
$$g_{Pxy}=i\tau^3, \quad g_{P_x}=i\tau^1, \quad g_{P_y}=i\tau^1, \quad g_T=\tau^0; \qquad (71)$$
$$g_{Pxy}=\tau^0, \quad g_{P_x}=\tau^0, \quad g_{P_y}=\tau^0, \quad g_T=i\tau^3; \qquad (72)$$
$$g_{Pxy}=\tau^0, \quad g_{P_x}=i\tau^3, \quad g_{P_y}=i\tau^3, \quad g_T=i\tau^3; \qquad (73)$$
$$g_{Pxy}=\tau^0, \quad g_{P_x}=i\tau^1, \quad g_{P_y}=i\tau^1, \quad g_T=i\tau^3; \qquad (74)$$
$$g_{Pxy}=i\tau^3, \quad g_{P_x}=\tau^0, \quad g_{P_y}=\tau^0, \quad g_T=i\tau^3; \qquad (75)$$
$$g_{Pxy}=i\tau^3, \quad g_{P_x}=i\tau^3, \quad g_{P_y}=i\tau^3, \quad g_T=i\tau^3; \qquad (76)$$
$$g_{Pxy}=i\tau^3, \quad g_{P_x}=i\tau^1, \quad g_{P_y}=i\tau^1, \quad g_T=i\tau^3; \qquad (77)$$
$$g_{Pxy}=i\tau^1, \quad g_{P_x}=\tau^0, \quad g_{P_y}=\tau^0, \quad g_T=i\tau^3; \qquad (78)$$
$$g_{Pxy}=i\tau^1, \quad g_{P_x}=i\tau^3, \quad g_{P_y}=i\tau^3, \quad g_T=i\tau^3; \qquad (79)$$
$$g_{Pxy}=i\tau^1, \quad g_{P_x}=i\tau^1, \quad g_{P_y}=i\tau^1, \quad g_T=i\tau^3; \qquad (80)$$
$$g_{Pxy}=i\tau^1, \quad g_{P_x}=i\tau^2, \quad g_{P_y}=i\tau^2, \quad g_T=i\tau^3; \qquad (81)$$
$$g_{Pxy}=i\tau^{12}, \quad g_{P_x}=i\tau^1, \quad g_{P_y}=i\tau^2, \quad g_T=\tau^0; \qquad (82)$$
$$g_{Pxy}=i\tau^{12}, \quad g_{P_x}=i\tau^1, \quad g_{P_y}=i\tau^2, \quad g_T=i\tau^3; \qquad (83)$$

In total, 272 possibilities

At most 196 different Z_2 spin liquids!

Wen, Phys. Rev. B **65**, 165113 (2002)

Fluctuations above the mean field and gauge fields

- Some results about lattice gauge theory (coupled to matter) may be used to discuss the stability/instability of a given mean-field Ansatz

- What is known about U(1) gauge theories?
  Monopoles proliferate $\longrightarrow$ confinement

  Polyakov, Nucl. Phys. B **120**, 429 (1977)

  Spinons are glued in pairs by strong gauge fluctuations and are not physical excitations

- Deconfinement may be possible in presence of gapless matter field
  The so-called U(1) spin liquid

  Hermele et al., Phys. Rev. B **70**, 214437 (2004)

- In presence of a charge-2 field (i.e., spinon pairing) the U(1) symmetry can be lowered to Z_2 $\longrightarrow$ deconfinement

  Fradkin and Shenker, Phys. Rev. D **19**, 3682 (1979)

- For example in D=2:
  - Z_2 gauge field (gapped) + gapped spinons may be a stable deconfined phase
    short-range RVB physics
    Read and Sachdev, Phys. Rev. Lett. **66**, 1773 (1991)
  - U(1) gauge field (gapless) + gapped spinons should lead to an instability towards confinement and valence-bond order
    Read and Sachdev, Phys. Rev. Lett. **62**, 1694 (1989)

Variational Monte Carlo for fermions

- The exact projection on the subspace with one spin per site can be treated within the variational Monte Carlo approach (part of the gauge fluctuations are considered!)

$$|\Phi\rangle = \mathcal{P}|\Phi_{MF}(U_{ij}^0)\rangle$$

- The variational energy

$$E(\Phi) = \frac{\langle\Phi|\mathcal{H}|\Phi\rangle}{\langle\Phi|\Phi\rangle} = \sum_x P(x)\frac{\langle x|\mathcal{H}|\Phi\rangle}{\langle x|\Phi\rangle}$$

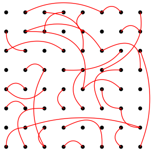
$P(x) \propto |\langle x|\Phi\rangle|^2$ and $|x\rangle$ is the (Ising) basis in which spins are distributed in the lattice

- $E(\Phi)$ can be sampled by using "classical" Monte Carlo, since $P(x) \geq 0$
- $\langle x|\Phi\rangle$ is a determinant
- The ratio of to determinants (needed in the Metropolis acceptance ratio) can be computed very efficiently, i.e., O(N), when few spins are updated
- The algorithm scales polynomially, i.e., $O(N^3)$ to have almost independent spin configurations

- The mean-field wave function has a BCS-like form

$$|\Phi_{MF}\rangle = \exp\left\{\frac{1}{2}\sum_{i,j} f_{i,j} c^{\dagger}_{i,\uparrow} c^{\dagger}_{j,\downarrow}\right\}|0\rangle$$

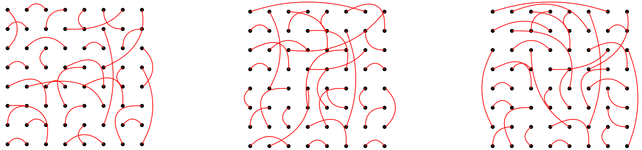
It is a linear superposition of all singlet configurations (that may overlap)



$+ \ldots$

- After projection, only non-overlapping singlets survive:
  the resonating valence-bond (RVB) wave function

Anderson, Science **235**, 1196 (1987)

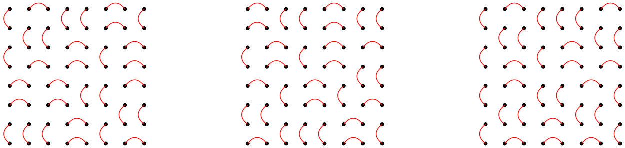


$+ \ldots$

- The mean-field wave function has a BCS-like form

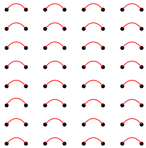
$$|\Phi_{MF}\rangle = \exp\left\{\frac{1}{2}\sum_{i,j} f_{i,j} c^{\dagger}_{i,\uparrow} c^{\dagger}_{j,\downarrow}\right\}|0\rangle$$

- Depending on the pairing function $f_{i,j}$, different RVB states may be obtained...



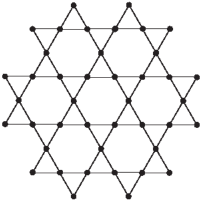
$+ \ldots$

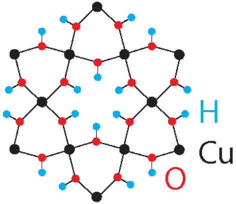
- ...even with valence-bond order (valence-bond crystals)



The Heisenberg model on the Kagome lattice

$$\mathcal{H} = J\sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + J' \sum_{\langle\langle ij \rangle\rangle} \mathbf{S}_i \cdot \mathbf{S}_j + \mathrm{DM} + \mathrm{distortions} + \mathrm{3D\ couplings} + \dots$$





- No magnetic order down to 50mK (despite $T_{CW} \simeq 200$K)
- Spin susceptibility rises with $T \to 0$ but then saturates below 0.5K
- Specific heat $C_v \propto T$ below 0.5K
- No sign of spin gap in dynamical Neutron scattering measurements

Mendels *et al.*, PRL 98, 077204 (2007)

Helton *et al.*, PRL 98, 107204 (2007)

Bert *et al.*, PRB 76, 132411 (2007)

Some of the previous results

Nearest-neighbor Heisenberg model on the Kagome lattice

Author	GS proposed	Energy/site	Method used
P.A. Lee	$U(1)$ gapless SL	$-0.42866(1)J$	Fermionic VMC
Singh	36-site HVBC	$-0.433(1)J$	Series expansion
Vidal	36-site HVBC	$-0.43221\ J$	MERA
Poilblanc	12- or 36-site VBC		QDM
Lhuillier	Chiral gapped SL		SBMF
White	Z_2 gapped SL	$-0.4379(3)J$	DMRG
Schollwoeck	Z_2 gapped SL	$-0.4386(5)J$	DMRG

Ran, Hermele, Lee, and Wen, PRL **98**, 117205 (2007)

Yan, Huse, and White, Science **332**, 1173 (2011)

$$S_i^\mu = \frac{1}{2} c^\dagger_{i,\alpha} \sigma^\mu_{\alpha,\beta} c_{i,\beta}$$

$$\mathcal{H} = -\frac{1}{2} \sum_{i,j,\alpha,\beta} J_{ij} \left(c^\dagger_{i,\alpha} c_{j,\alpha} c^\dagger_{j,\beta} c_{i,\beta} + \frac{1}{2} c^\dagger_{i,\alpha} c_{i,\alpha} c^\dagger_{j,\beta} c_{j,\beta} \right)$$

$$c^\dagger_{i,\alpha} c_{i,\alpha} = 1 \qquad c_{i,\alpha} c_{i,\beta} \epsilon_{\alpha\beta} = 0$$

- At the mean-field level:

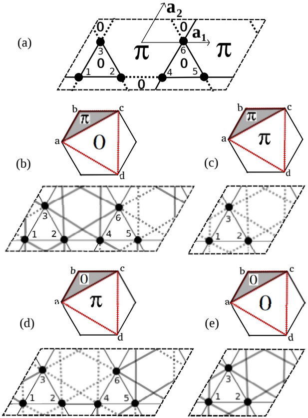
$$\mathcal{H}_{\rm MF} = \sum_{i,j,\alpha} (\chi_{ij} + \mu \delta_{ij}) c^\dagger_{i,\alpha} c_{j,\alpha} + \sum_{i,j} (\eta_{ij} + \zeta \delta_{ij}) c^\dagger_{i,\uparrow} c^\dagger_{j,\downarrow} + h.c.$$

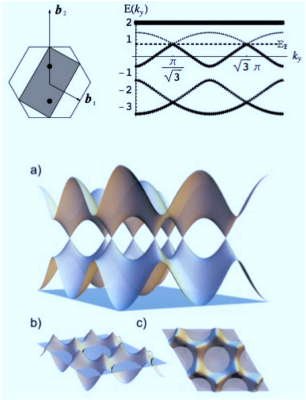
$$\langle c^\dagger_{i,\alpha} c_{i,\alpha} \rangle = 1 \qquad \langle c_{i,\alpha} c_{i,\beta} \rangle \epsilon_{\alpha\beta} = 0$$

- Then, we reintroduce the constraint of one-fermion per site:

$$|\Phi(\chi_{ij}, \eta_{ij}, \mu)\rangle = \mathcal{P}_G |\Phi_{\rm MF}(\chi_{ij}, \eta_{ij}, \mu, \zeta)\rangle$$

$$\mathcal{P}_G = \prod_i (1 - n_{i,\uparrow} n_{i,\downarrow})$$





- The U(1) gapless (Dirac) spin liquid is a good variational Ansatz

Ran, Hermele, Lee, and Wen, PRL **98**, 117205 (2007)

- It is stable for dimerization

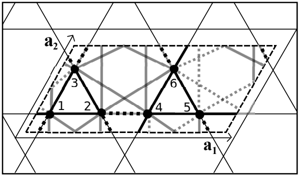
Iqbal, Becca, and Poilblanc, PRB 83, 100404 (2011); New Journal of Phys., to appear

Can we have a Z_2 gapped spin liquid (DMRG)?

Projective symmetry-group (PSG) analysis

Lu, Ran, and Lee, PRB **83**, 224413 (2011)

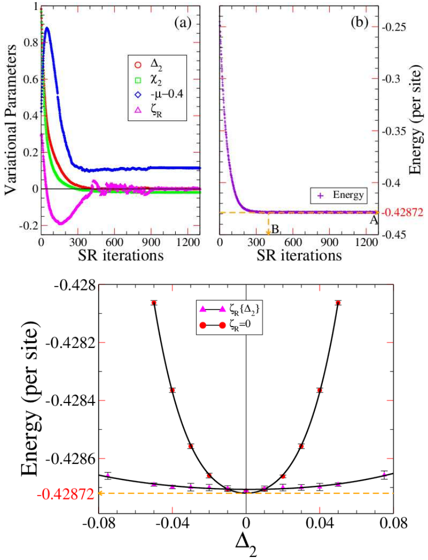
$$U_{ij}^0 = \begin{pmatrix} \chi_{ij} & \eta_{ij}^* \\ \eta_{ij} & -\chi_{ij}^* \end{pmatrix}$$



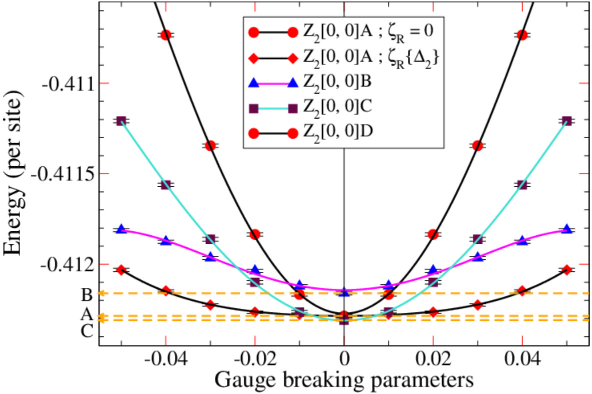
No.	η_{12}	Λ_s	u_α	u_β	u_γ	$\tilde{u}_\gamma$	Label	Gapped?
1	+1	τ^2,τ^3	τ^2,τ^3	τ^2,τ^3	τ^2,τ^3	τ^2,τ^3	$Z_2[0,0]A$	Yes
2	−1	τ^2,τ^3	τ^2,τ^3	τ^2,τ^3	τ^2,τ^3	0	$\mathbb{Z}_2[0,\pi]\beta$	**Yes**
3	+1	0	τ^2,τ^3	0	0	0	$Z_2[\pi,\pi]A$	No
4	−1	0	τ^2,τ^3	0	0	τ^2,τ^3	$Z_2[\pi,0]A$	No
5	+1	τ^3	τ^2,τ^3	τ^3	τ^3	τ^3	$Z_2[0,0]B$	Yes
6	−1	τ^3	τ^2,τ^3	τ^3	τ^3	τ^2	$Z_2[0,\pi]\alpha$	No
7	+1	0	0	τ^2,τ^3	0	0	–	–
8	−1	0	0	τ^2,τ^3	0	0	–	–
9	+1	0	0	0	τ^2,τ^3	0	–	–
10	−1	0	0	0	τ^2,τ^3	0	–	–
11	+1	0	0	τ^2	τ^2	0	–	–
12	−1	0	0	τ^2	τ^2	0	–	–
13	+1	τ^3	τ^3	τ^2,τ^3	τ^3	τ^3	$Z_2[0,0]D$	Yes
14	−1	τ^3	τ^3	τ^2,τ^3	τ^3	0	$Z_2[0,\pi]\gamma$	No
15	+1	τ^3	τ^3	τ^3	τ^2,τ^3	τ^3	$Z_2[0,0]C$	Yes
16	−1	τ^3	τ^3	τ^3	τ^2,τ^3	0	$Z_2[0,\pi]\delta$	No
17	+1	0	τ^2	τ^3	0	0	$Z_2[\pi,\pi]B$	No
18	−1	0	τ^2	τ^3	0	τ^3	$Z_2[\pi,0]B$	No
19	+1	0	τ^2	0	τ^2	0	$Z_2[\pi,\pi]C$	No
20	−1	0	τ^2	0	τ^2	τ^3	$Z_2[\pi,0]C$	No

Only ONE gapped SL connected with the U(1) Dirac SL: The $Z_2[0,\pi]\beta$ spin liquid
FOUR gapped SL connected with the Uniform U(1) SL: $Z_2[0,0]$A, B, C, and D

The Dirac U(1) SL is stable against opening a gap...



...and also the Uniform U(1) spin liquid is stable



The gapless U(1) Dirac SL is very stable

- Against dimerization
- For breaking the gauge structure down to Z_2

The gapless uniform U(1) SL is stable against Z_2 SLs